Cosmic ray anisotropies:
The role of perpendicular diffusion

Du Toit Strauss \textsuperscript{12} Horst Fichtner \textsuperscript{1}

\textsuperscript{1}Institute for Theoretical Physics: TP 4
Ruhr University Bochum
Germany

\textsuperscript{2}Centre for Space Research
North-West University
South Africa

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Introduction and motivation ...
We study CR transport in the heliosphere

The heliosphere ...

... and resulting CR intensities

Using insights from global MHD models, localized turbulence models, particle scattering theories and different datasets and simulations for each of these steps, we simulate the propagation of CRs in the heliosphere and compare these with (mainly) spacecraft observations.
We have, in the past, very successfully applied the Parker transport equation for CR modulation

\[
\frac{\partial f}{\partial t} = -(\vec{u} + \vec{v}_d) \cdot \nabla f + \nabla \cdot (K \cdot \nabla f) + \frac{1}{3} (\nabla \cdot \vec{u}) \frac{\partial f}{\partial \ln P}
\]

This is only valid for a \textit{(nearly?)} isotropic CR distribution

But, some observation show that we need to start working on the pitch-angle level ...
CRs at the TS, e.g. Decker et al.

Jovian electrons, e.g. Dunzlaff et al.

Cosmic ray anisotropies

Strauss and Fichtner (RUB, CSR)
CRs at the heliopause, e.g. Krimigis et al.

SEPs, e.g. Dröge et al.

- FD solution, 80 keV
- $\lambda_r = 0.12$ AU, const
- $t_i = 02:30$ UT

Cosmic ray anisotropies
We consider a “reduced” version of the Skilling equation:

\[
\frac{\partial f(\vec{x}, \mu, t)}{\partial t} = -\nabla \cdot (\mu \nu \hat{b}f) - \frac{\partial}{\partial \mu} \left( \frac{1 - \mu^2}{2L} \nu f \right) + \frac{\partial}{\partial \mu} \left( D_{\mu\mu} \frac{\partial f}{\partial \mu} \right) + \nabla \cdot (D_\perp \cdot \nabla f)
\]

which neglects e.g. convection and energy losses.

The focus is on emphasizing the importance (effects) of perpendicular diffusion for (i) solar energetic particles and (ii) cosmic rays near the heliopause

A small caveat on numerics...

All previous FD solutions of the focussed TPE did not conserve particles!

- We can by applying a LOD numerical scheme
- E.g. for $\mu$ two eqs are solved: One for the focussing part (first-order; hyperbolic), another for the diffusion part (second-order; parabolic)
- We don’t use “traditional” boundary conditions – the fluxes into and out-of the last grid cell (on a staggered grid) is evaluated
E.g. for the $\mu$-dimension

\[
\frac{1}{6} \frac{\partial f}{\partial dt'} = -\frac{\partial}{\partial \mu} \left( \frac{1 - \mu^2}{2L} v_f \right)
\]

\[
\frac{1}{6} \frac{\partial f}{\partial dt'} = \frac{\partial D_{\mu\mu}}{\partial \mu} \frac{\partial f}{\partial \mu} + D_{\mu\mu} \frac{\partial f^2}{\partial \mu^2}
\]

We end up with 6 equations (1 advective and 1 diffusive equation for each of the 3 dimensions). Advection eqs are solved by an upwind scheme, diffusion eqs by an explicit scheme.

At the boundaries, we consider the net-fluxes,

\[
f_{i=N}^{t+\Delta t} = f_{i=N}^{t} + \frac{\Delta t}{\Delta \mu} F_{i=N-1/2}^{t}
\]

where $F_{i=N-1/2}^{t}$ is the (advective or diffusive) flux entering or leaving the last cell.
A comparison with Effenberger & Litvinenko (2014)
Solar energetic particles ...
- The observed distribution at Earth’s orbit (1 AU)
- Is the distribution symmetric in terms of longitude?
- Is the peak intensity at the point of optimal magnetic connection between source and observer?
The modelled distribution is not symmetric about $\pm \phi$
The intensity for different levels of perpendicular diffusion

The distribution is not a symmetrical Gaussian – the intensities are higher towards the west of optimal magnetic connection

The peak intensity is also shifted – again towards the west of optimal magnetic connectivity

Does this make sense??
Yes, because ...

Although advection is directed along the HMF – creating a distribution symmetric along 1 AU – perpendicular diffusion acts perpendicular to the mean field – setting up a distribution symmetric perpendicular to the mean field.

The peak intensity can be shifted along $\phi$ when the diffusion coefficients in the global coordinate system has non-zero derivatives, $\nabla \cdot \mathbf{D}_\perp \neq 0$ – creating a convection term.
This is governed by $D_\perp$. But what is this coefficient?

Qin et al. (2013): Pitch-angle dependence doesn’t matter – $D_\perp$ independent of $\mu$

Dröge et al. (2010):  
$D_\perp \propto v_\perp \propto \sqrt{1 - \mu^2}$

FLRW, e.g. Jokipii (1966):  
$D_\perp \propto v_\parallel \propto |\mu|$

To us, this dependence is mostly neglected in the literature

We use these three forms, but normalized such that  
$K_\perp = \frac{1}{2} \int_{-1}^{+1} D_\perp(\mu)d\mu$ are all the same
Significant effect even for relatively weak levels of perpendicular diffusion
The pitch-angle dependence can lead to even more interest results when examining a different application ...

Note: There is a problem with causality!
So, consider the transport equation in the limiting case of $\mu = \pm 1$

$$\mu = \pm 1 : \frac{\partial f}{\partial t} = \mp v \frac{\partial f}{\partial z} + \frac{\partial}{\partial x} \left( D_\perp \frac{\partial f}{\partial x} \right)$$  \hspace{1cm} (4)

For the FLRW model, $D_\perp (\mu = \pm 1) \neq 0!$

- Streaming along $\vec{B}$ displaces particles by $\Delta z = \pm vt$
- Diffusion perpendicular to $\vec{B}$ (for FLRW) displaces particles by $\Delta x \neq 0$

The net displacement of a particle is thus

$$\Delta s = \sqrt{(\Delta z)^2 + (\Delta x)^2} > vt$$  \hspace{1cm} (5)

There seems to be some discrepancy between the transport equation and the FLRW formulation. This is not observed in the way FLRW is incorporated by Laitinen et al. (2013)
Cosmic ray intensities near the heliopause . . .
The CR intensities across the HP was of the expected form: Galactic CR intensities increase and heliospheric CRs (termination shock and anomalous particles) decrease...

Stone et al. (2013)
Krimigis et al. (2013) observes anisotropic CR distributions

Inside the inner heliosheath, the distributions are isotropic
In the interstellar medium, all distributions show anisotropies

Heliospheric particles: Distribution peaks at 90° pitch
Galactic particles: Distribution is a minimum at 90° pitch
Further assumptions:

- HP located at $x = 0$
- Galactic particles specified at all blue boundaries
- Heliospheric particles specified at all red boundaries
- Inside HP, $D_{\mu\mu}$ is large (a lot of scattering; $\lambda_\parallel \sim 1$ AU)
- Beyond the HP: $D_{\mu\mu}$ is small (not so much scattering; $\lambda_\parallel \sim 1000$ AU)
- We look at a cut along $y = 0$
Further assumptions about the $\mu$ dependence:

- $D_{\mu\mu}$ follows normal QLT dependence

$D_{\perp}$ is however less well known:

- FLRW gives $D_{\perp} \sim v_\parallel \sim |\mu|$
- But, $D_{\perp} \sim v_\perp \sim \sqrt{1 - \mu^2}$ is also an option?

The latter formulation is used in the study of SEPs (e.g. Dröge et al., 2010)

- Could also result from a random drift process when $r_L \ll l_c$ (see Fraschetti & Jokipii, 2011)
- Intensities at different pitch-angles
“Comparison” with observed anisotropies

- Isotropy inside the heliosphere ($\lambda_|| \ll L$) and anisotropic behaviour in interstellar medium ($\lambda_|| \gg L$)
To summarize ...
We have shown for SEPs that:

- By including perpendicular diffusion, the resulting distribution at 1 AU is not symmetric in $\phi$ and the peak intensity may be shifted away from direct magnetic connection – see also the papers by He et al., Dresing et al., Lario et al.....

- The pitch-angle dependence of $D_\perp$ seems to be an important and mostly ignored transport parameter.

- There is a (big?) causality problem related to the implementation of the FLRW $D_\perp$.

While, for CRs at the HP:

- Adopting the appropriate form of $D_\perp(\mu)$ can simultaneously reproduce the anisotropies observed for galactic and heliospheric particles.
Outstanding questions and model developments we are busy with:

- What is the “source function” for SEPs?
  \[ \Rightarrow \] We need: (i) temporal profile, (ii) energy spectrum and (ii) to account for coronal transport

- What processes are responsible for the perpendicular transport of SEP?
  \[ \Rightarrow \] Diffusion probably plays a large role, but \( D_\perp \) is not well understood
  \[ \Rightarrow \] Implementing more realistic transport coefficients, based on the underlying turbulence

- What about wave generation (a self-consistent coupling)?
  \[ \Rightarrow \] These electrons sample the dissipation range, which might give interesting results
Thank you ...