EVOLUTION OF SOLAR WIND FLUCTUATIONS AND THE INFLUENCE OF TURBULENT 'MIXING'

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ABSTRACT

We present various numerical and analytical solutions for the transport of solar wind turbulence. The model used takes into account the effects of convection, expansion, and wave propagation, as well as the recently illuminated effects of (non-WKB) 'mixing' terms. The radial evolution of the fluctuating kinetic energy (E_k), magnetic energy (E_b) and normalized cross helicity (σ_c) is computed, and, it is demonstrated that in appropriate limits the solutions converge to the WKB forms. In the general case, solutions which differ substantially from those predicted by WKB theory are obtained. The degree of turbulent 'mixing' shows considerable dependence on the nature of the turbulence, giving rise to varying levels, at 1AU, of the ratio of "inward" and "outward" fluctuation energies and the ratio of kinetic to magnetic energies in the fluctuations. The transport properties described here may provide at least a partial explanation for the observed mixing of cross helicities with increasing heliocentric distance in the solar wind.

INTRODUCTION AND THE MODEL

The problem of how to adequately describe the physics of fluctuations of the interplanetary medium has been present since the earliest spacecraft observations showed that such fluctuations are ubiquitous in the solar wind. Recently developed theories of the transport of MHD scale turbulence in a weakly inhomogeneous background plasma provide a basis for computing both radial and temporal dependence of the spectrum of solar wind fluctuations /1/. Here we report on the results of a numerical and analytic investigation of such a transport model. As a consequence of the length constraints to which this paper is subject we are of necessity concise in our discussion, and we refer the reader to previous and forthcoming publications for further details *e.g.*, /1,2,3,4,5/, and also to the companion articles of Grappin, Mangeney & Velli; Marsch; Tu; Velli, Grappin & Mangeney, and in particular, Matthaeus *et.al.* appearing in this volume.

The interplanetary medium is assumed to be a single component magnetofluid obeying the usual compressible MHD fluid equations. A two length (and time) scale decomposition of these dynamical equations is performed, in which the the fields depend upon both large ($\mathbf{R}$) and small ($\mathbf{x}$) scale spatial co-ordinates. Thus each field separates into two components: (1) a spatially slowly varying 'mean' part, depending only on the large-scale, and (2) a fluctuating portion which depends on both the large and small spatial scales. On the basis of observational evidence (*e.g.*, /6,7/), and also for simplicity, we assume that the small-scale fluctuations are both incompressible and homogeneous. Hence, the only fluctuating quantities are the velocity ($\mathbf{v}$) and magnetic field ($\mathbf{b}$). Note that the large scale fields are *not* required to be either homogeneous or incompressible.

Straightforward algebraic manipulations yield a set of coupled transport equations for such physically important correlation functions as $S_{ij}^{vb}(\mathbf{R},\mathbf{r}) = < v_i(\mathbf{R},\mathbf{x})\, b_j(\mathbf{R},\mathbf{x}+\mathbf{r}) >$, where the angle

brackets denote averaging over $\mathbf{x}$ /1/. These equations contain terms involving the *large*-scale slowly varying fields, which we take to be specified. As a result of both the symmetries and the simple physical interpretation associated with the Elsässer variables ($\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{b}/\sqrt{4\pi\rho}$), it proves convenient to use this 'inward' and 'outward' propagating modes representation. Finally we Fourier transform the equations with respect to the separation parameter $\mathbf{r}$ (conjugate variable $\mathbf{k}$).

If we denote the energy in the Elsässer fields by $P^{\pm}(\mathbf{R}, k_r)$, the energy difference (residual energy /8/) by $F \propto E_k - E_b$, and the helicity of the induced electric field by $J \propto Im\{\mathbf{v}^*(\mathbf{k}) \cdot \mathbf{b}(\mathbf{k})\}$, then the final set of equations is:

$$\frac{\partial P^{\pm}}{\partial t} + (U \mp V_{Ar})\frac{\partial P^{\pm}}{\partial R} + \left(\frac{U \pm V_{Ar}}{R}\right)P^{\pm} + M^{\pm}_{ki}F_{ik}$$

$$\frac{\partial F}{\partial t} + U\frac{\partial F}{\partial R} + \frac{U}{R}F - (2\mathbf{k}\cdot\mathbf{V}_A)\,J + 2\left[M^{+}_{ki}P^{-}_{ik} + M^{-}_{ki}P^{+}_{ik}\right] = NL^F$$

$$\frac{\partial J}{\partial t} + U\frac{\partial J}{\partial R} + \frac{U}{R}J + (2\mathbf{k}\cdot\mathbf{V}_A)\,F = NL^J$$

where U is the constant, radially directed mean wind speed, $\mathbf{B}_0$ is the mean magnetic field—taken equal to the standard Parker spiral, $\rho \propto 1/R^2$, is the large-scale fluid density, $\mathbf{V}_A = \mathbf{B}_0/\sqrt{4\pi\rho}$, is the large-scale Alfvén velocity, and we refer to $M^{\pm}_{ki}$ as the 'mixing' operators. In order to facilitate comparisons with observations we take $\mathbf{r}$ to be in the radial direction, the spectra then being reduced ones (*i.e.,* functions of k_r rather than $\mathbf{k}$). It should be stressed that we have made *no* approximations regarding the relative abundances of the 'inward' and 'outward' modes. In fact the model supports *completely* arbitrary admixtures of these modes. The terms on the left of the equations are all linear and represent the effects of convection, expansion, wave propagation and 'mixing', while those on the right represent non-linear interactions. Note that in contrast to the case of WKB transport *all* of these effects, including 'mixing', are present at *leading* order. In this preliminary study we focus upon the properties of the linear transport operators, dropping hereafter all non-linear terms*.

Before moving on to the results, a few words are in order regarding the nature of the 'mixing' operators. Physically, we can interpret 'mixing' as a scattering of the $\mathbf{z}^{\pm}$ modes due to large-scale gradients of the mean fields. The operators have the form:

$$M^{\pm}_{ji}(\mathbf{R}) = \frac{\partial \mathbf{U}_i}{\partial \mathbf{R}_j} \pm \frac{1}{\sqrt{4\pi\rho}}\frac{\partial \mathbf{B}_{0i}}{\partial \mathbf{R}_j} - \frac{1}{2}\delta_{ij}\nabla\cdot\left(\frac{\mathbf{U}}{2} \pm \mathbf{V}_A\right)$$

which is completely determined by the the *large*-scale gradients of the *mean* fields. However, because the operators always appear coupled to *small*-scale spectral tensors (*e.g.,* $Q_{nj}M^{\pm}_{ji}$), 'mixing' also depends on the nature and rotational symmetry properties of the small-scale turbulence. Assuming that the small-scale turbulence is either isotropic, slab, or two-dimensional (2-D) enables the trace of $Q_{nj}M^{\pm}_{ji}$ to be evaluated and written as $M^{\pm}Q$, where $Q = Q_{ii}$, and the $M^{\pm}$ are effective 'mixing' operators. For values of $R \gtrsim 2AU$ these effective operators are all essentially the same. Inside $1AU$, however, important differences exist between both the plus and minus versions for the same type of turbulence, and also between $M^{\pm}$ for different types of rotational symmetry.

The impact of the 'mixing' term on the radial evolution of the physical quantities is crucially affected by the size of $\mathbf{k}\cdot\mathbf{V}_A$. This factor is the coupling strength between the F and J fields and may be considered as a 'WKB enforcing' term. If $\mathbf{k}\cdot\mathbf{V}_A \approx 0$, then strong mixing occurs, since the initial dominance of the 'outward' mode causes growth of F, which in turn causes growth of the 'inward' mode. Thus $\sigma_c = (P^- - P^+)/(P^- + P^+)$ decreases significantly with heliocentric distance. However, when F and J are strongly coupled, the energy in the 'inward' fluctuations remains a tiny fraction of that in the 'outward' and WKB-like solutions are obtained.

*Note that for fully developed turbulence there is no net spectral transport of $P^{\pm}$ in the inertial range, *i.e.,* $NL^{\pm} = 0$.

RESULTS AND DISCUSSION

In order to solve the equations we have, in most cases, resorted to numerical techniques, namely Chebyshev spectral (collocation) methods /9,10/, where the equations are integrated in time to steady-state solutions. We choose to impose the boundary conditions on the fields at R_0 ($\equiv 10R_{sun}$) so that the fluctuations there are *purely* outwardly propagating. This allows the inner boundary to be interpreted as the Alfvén radius, *i.e.*, the distance at which the (radial) flow velocity becomes equal to the (radial) Alfvén velocity. Chebyshev techniques were chosen to facilitate the subsequent inclusion of non-linear terms, and also because they support arbitrary boundary conditions.

The first case we consider is that of *isotropically* distributed fluctuations. The power-law ($k^{-\alpha}$) inertial range turbulence is characterised by a single parameter, namely its spectral slope α. Typically we choose values corresponding to the Kolmogorov ($\alpha = 5/3$) or Kraichnan ($\alpha = 3/2$) scenarios. It can be shown that for isotropic symmetries J is identically zero. An analytic solution to the $\mathbf{V}_A = 0$ version of these equations was presented by Zhou & Matthaeus /1/, and this provided a useful test of our numerical accuracy. Since J is explicitly zero, the WKB enforcing term cannot come into play, and thus we see a substantial falloff in the normalized cross helicity with increasing heliocentric distance. The solutions show significant dependence on (a) the spectral index α: increasing values causing faster radial decay of σ_c; and (b) the value of $A_0 = V_{Ar}(R_0)/U$, where V_{Ar} is the radial component of the large-scale Alfvén velocity (Figure 1a). This latter dependence decreases the effect of 'mixing' as A_0 is increased from zero to unity. The case of $A_0 = 1$ corresponds to R_0 being the Alfvén (critical) radius, while smaller positive values may be interpreted as R_0 exceeding this radius. In such cases we still enforce purely outward fluctuations at the inner boundary, despite the fact that this is no longer physically necessary. The $A_0 = 0$ case represents the situation in the absence of a large-scale magnetic field, *e.g.*, within the current sheet.

The second case relates to *slab* geometry, where the fluctuations are in the plane perpendicular to the wave-vector $\mathbf{k}$. The direction of $\mathbf{k}$ is taken to be parallel to either $\mathbf{R}$ or $\mathbf{B}_0$, the results being similar for both cases. Strong 'mixing' is seen in the separate cases of $\mathbf{k} = 0$ and $\mathbf{V}_A = 0$. However as the radial component of $\mathbf{k}$ is increased towards a reciprocal correlation scale, the results rapidly return to WKB-like solutions. As mentioned above this is because non-zero $\mathbf{k} \cdot \mathbf{V}_A$ means that F and J are tightly coupled. Such coupling constrains F to oscillate about zero (*i.e.*, remain small), and thus, since P^+ is driven only by F at this order, 'mixing' is strongly inhibited (Figure 1a).

Finally we consider 2-D turbulence, by which we mean (a) $\mathbf{k} \perp \mathbf{B}_0$, (b) fluctuations are perpendicular to both $\mathbf{k}$ and $\mathbf{B}_0$, and (c) fluctuations are distributed isotropically (in the planes normal to $\mathbf{B}_0$) with a power-law inertial range. As a consequence of this geometry $\mathbf{k} \cdot \mathbf{V}_A \equiv 0$, ensuring that

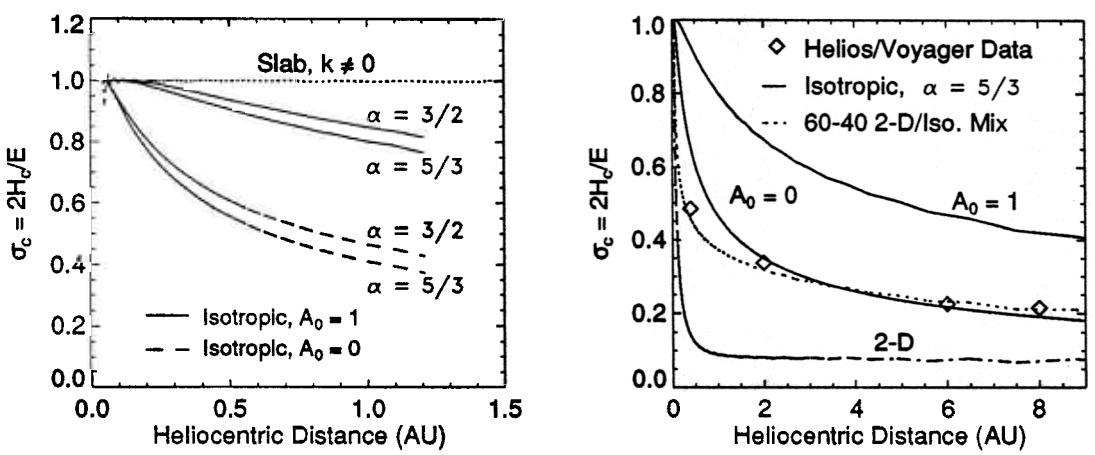

Fig. 1. Plots of the numerical solutions for the normalized cross helicity as a function of distance. (a) isotropic and slab solutions, (b) comparison with data.

F and J are decoupled, and thence that 'mixing' is always strong. In fact, J is identically zero for reasons which are essentially the same as those applying in the fully isotropic case. Significant dependence on the values of the spectral index α, and A_0 is again seen. Simulation results /11/ have shown that in the presence of a mean magnetic field, energy is transferred to the $\perp$ components of the fluctuations much more rapidly than it is to the $\|$ ones. In other words the 2-D spectrum 'switches on' first. This suggests that 2-D fluctuations may be particularly relevant to the solar wind system, and indeed there is also some observational evidence that the solar wind can be modeled as an admixture of slab ($\mathbf{k} \| \mathbf{B}_0$) and 2-D fluctuations /12/. To this end we show in Figure 1b several numerical solutions for the normalized cross helicity with observational data points from Helios and Voyager data superimposed (3 hour averages). Without attempting to optimise the agreement between theory and data, we note that the 60% 2-D, 40% isotropic mixture provides a remarkably good fit to the data. We are not suggesting that this is in fact the state of affairs in the solar wind, only that the theory clearly shows strong potential to explain the observations.

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