Measurement of Spectral Anisotropy Using Single Spacecraft Data

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Abstract. A technique for analyzing multiple-time, single-point datasets is presented, which yields information about the (spectral) variation of MHD fields in directions perpendicular to the mean flow. The analysis can be performed for any homogeneous solenoidal fields, including magnetic and incompressible velocity fluctuations. A summary of the associated theory is given, along with initial results obtained using an interval of Voyager data.

INTRODUCTION

Almost all *in situ* observations of solar wind and magnetospheric parameters are obtained from measurements using one spacecraft at a time. Associated datasets usually consist of measurements of magnetic field **B**, and plasma velocity **V** and density n, obtained at regular time intervals along the spacecraft trajectory. These measurements can be used to construct correlation functions and spectra, *e.g.*, (4, 11), and the related analysis has added immensely to our understanding of fluctuations at MHD scales.

Nonetheless, because data intervals are approximately equivalent to simultaneous measurements at discrete points along a curve in space (see below), there are strong constraints on the extractable information. For example, only limited information regarding gradients in **B** oblique to the measurement curve can be obtained. Recently we have shown that suitably defined mean wavenumbers can be calculated in directions *perpendicular* to the reduction (measurement) direction, for certain fields provided that the associated fluctuations are solenoidal (18).

The approach is an expansion of the technique, introduced in (15), for extracting the magnetic helicity from one-point, multiple-time datasets. Thus, even though only reduced spectra are available, it is still possible to obtain information about the scale-lengths characterizing the fluctuations in directions transverse to the measurement direction. In the following section we summarize the associated theory and apply it to several types of fluctuations believed to be important in the solar wind (16). An interval of Voyager 2 data is then analyzed in this context.

THEORY

General forms for the correlation functions and spectra associated with homogeneous incompressible MHD turbulence have been derived elsewhere. See (18) for a summary/review. For example, if $\langle\cdots\rangle$ is an appropriate averaging procedure, and $\mathbf{b}(\mathbf{x}) = \mathbf{B} - \langle\mathbf{B}\rangle$ is the fluctuating magnetic field, then the correlation function for the magnetic fluctuations is $R_{ij}(\mathbf{r}) = \langle b_i(\mathbf{x})\, b_j(\mathbf{x}+\mathbf{r})\rangle$. Taking the Fourier transform yields the spectral tensor $S_{ij}(\mathbf{k})$, which consists of index symmetric (I_{ij}) and antisymmetric (J_{ij}) pieces. The former is determined by three scalar functions (18) and contains, among other contributions, the magnetic energy spectrum which has been studied extensively in the solar wind context (7, 22). The antisymmetric portion is controlled by a single scalar function $H(\mathbf{k})$, proportional to the magnetic helicity spectrum, and has the specific form (1, 15, 19)

$$J_{ij} = i\epsilon_{ij\alpha}k_\alpha H(\mathbf{k}). \tag{1}$$

Thus, if one could simultaneously sample $\mathbf{B}(\mathbf{x},t)$ at many locations throughout a given volume, then $R_{ij}(\mathbf{r})$ and hence $J_{ij}(\mathbf{k})$ could be estimated. Unfortunately, this is not achievable using data from a single spacecraft, which is the usual situation for the solar wind. Instead, one obtains values of **B** at a sequence of times corresponding to positions along the spacecraft trajectory. However, as the mean flow speed $U\hat{\mathbf{R}}$ is much greater than (a) the spacecraft velocity, and (b) typical turbulent and wave velocities, one can invoke Taylor's "frozen-flow" hypothesis (8, 21) to relate the time series at a single point to a

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spatial sampling in the flow direction at a fixed time: $\mathbf{b}(\mathbf{x} = 0, t_j) \equiv \mathbf{b}(-Ut_j\hat{\mathbf{R}}, 0)$, where $\hat{\mathbf{R}}$ is the unit vector for the heliocentric radius. As a consequence only *reduced spectra*, rather than the more informative and desirable full wavevector spectra, are available (1, 5, 11). For Cartesian coordinate systems with the x-direction aligned with the radial (reduction) direction, one can show that the antisymmetric component of the reduced spectral tensor is

$$J_{ij}^{\text{red}}(k_1) \quad = \quad \int \mathrm{d}k_2 \mathrm{d}k_3 \, J_{ij}(k_1, k_2, k_3), \qquad (2)$$

so that contributions from all directions perpendicular to the reduction one are integrated out. Substituting from Eq. (1) we have $J_{12}^{\text{red}} \sim \int k_3 H(\mathbf{k})$. This is a quantity that depends upon the structure of the helicity spectrum in the $\hat{\mathbf{z}}$ direction, but according to Eq. (2) it can be evaluated from information obtained from the frozen-in condition in the $\hat{\mathbf{x}}$ direction! It is tempting, qualitatively, to interpret J_{12}^{red} as a mean value for k_3, weighted by the helicity spectrum. Caution is warranted, however, since the helicity cannot strictly speaking be a density or weight function as it is not a positive definite quantity. Hence, we define

$$\overline{k}_3(k_1) \quad = \quad \frac{\int k_3 H(\mathbf{k}) \, \mathrm{d}k_2 \mathrm{d}k_3}{\int H(\mathbf{k}) \, \mathrm{d}k_2 \mathrm{d}k_3} = k_1 \frac{J_{12}^{\text{red}}(k_1)}{J_{23}^{\text{red}}(k_1)}, \quad (3)$$

and similarly $\overline{k}_2 = k_1 J_{31}^{\text{red}} / J_{23}^{\text{red}}$.

As the $\overline{k}$ are functions of the reduced wavenumber k_1 (component of $\mathbf{k}$ in the measurement direction), we obtain helicity-weighted mean wavenumbers at *each* reduced wavenumber. One can also integrate over k_1 to obtain "bulk" wavenumbers in the $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ directions, *e.g.*, $\overline{K}_3 = \int \overline{k}_3(k_1) \, \mathrm{d}k_1$, which provide estimates for the transverse lengthscales characterizing the large-scale (helical) structures present in the fluctuations.

The above quantities are readily computed from suitable datasets and can be employed to provide previously overlooked or neglected information on the structure and scales of solar wind fluctuations.

In view of $H(\mathbf{k})$ not being positive definite, cancellation can occur in the averaging. This means that right-handed and left-handed structures at nearly equal scales give cancelling contributions, which tend to push the net "mean wavenumber" towards zero. It would perhaps be more useful to have access to $\int k_3 |H(\mathbf{k})| \, \mathrm{d}k_2 \mathrm{d}k_3$, *etc*, but this does not appear to be achievable using single-spacecraft datasets (see, however, the definitions for $\overline{K}_j$).

Special Cases. When the turbulence has a particular symmetry the $\overline{k}_j$ can often be evaluated further. Here we are interested in the isotropic, 2D, $2\frac{1}{2}$D, and

Table 1. Mean wavenumbers for specific fluctuation symmetries in the Cartesian coordinate system with $\hat{\mathbf{x}}$ in the radial direction and $\mathbf{B}_0$ in the $\hat{\mathbf{x}}$–$\hat{\mathbf{y}}$ plane.

	isotropic	2D	$2\frac{1}{2}$D	slab
$\overline{k}_2(k_1)$	arbitrary	0		$k_1 \tan\psi$
$\overline{k}_3(k_1)$	but equal	0		0

slab cases. For most of these symmetries we assume the existence of a well-defined mean magnetic field $\mathbf{B}_0 = \langle \mathbf{B} \rangle$. This preferred direction makes some coordinate systems more convenient to work with, and here we choose $\hat{\mathbf{x}} \equiv \hat{\mathbf{R}}$, with $\mathbf{B}_0$ in the x–y plane at an angle ψ to $\hat{\mathbf{R}}$. Further details on the full correlation and spectral tensors connected with these symmetries are given in (18), for example.

In the absence of any preferred directions one expects on physical grounds that the fluctuations will be isotropic, for which case it is easily shown that $\overline{k}_2 = \overline{k}_3$. Strictly 2D fluctuations, *i.e.*, those with wavevectors *and* Fourier amplitudes perpendicular to $\mathbf{B}_0$, have $H(\mathbf{k}) = 0$, so that the $\overline{k}_j$ also vanish.

The related case of $2\frac{1}{2}$D symmetry ($\mathbf{k} \cdot \mathbf{B}_0 = 0$ but no restriction on amplitude direction) can be shown to have $\overline{k}_3(k_1)$ arbitrary, but $\overline{k}_2(k_1) = k_1 \cot\psi$ independent of $H(\mathbf{k})$. The situation is similar for slab (parallel propagating Alfvén wave) fluctuations: $\overline{k}_3 = 0$ and $\overline{k}_2 = k_1 \tan\psi$. These results are summarized in Table 1. It is apparent that there is considerable scope for using the $\overline{k}$'s as diagnostics for distinguishing between the symmetry states of the fluctuations present in the solar wind.

DATA ANALYSIS AND RESULTS

To illustrate the above technique for calculating mean wavenumbers $\overline{k}_j$ we have analyzed an interval of Voyager 2 data from days 95–98 of 1978. At this time the spacecraft was near 2.8 AU, and experiencing a reasonably stable mean magnetic field, with $\psi \approx 60°$ and $B_0 \approx 24$ km/s in Alfvén speed units. The interval is a subset of one used in (11). The data points consist of 96 second averages for $\mathbf{V}$, $\mathbf{B}$, and n, with the entire set being rotated to the above discussed coordinate system prior to analysis. Correlation functions and spectra were computed using the procedure described in (11), based on Blackman–Tukey mean lagged product calculations of R_{ij} with 20 degrees of freedom. While the stationarity and homogeneity of solar wind fluctuations has not been rigorously established, empirically it appears that when the traced correlation function $R_{\alpha\alpha}(r)$ is of Lanczos type [that is, for some L, $R_{\alpha\alpha}(r > L)/R_{\alpha\alpha}(0) \ll 1$] then weak homogeneity is an acceptable assumption (11, 12). For the interval considered here $R_{\alpha\alpha}(r)$ approaches Lanczos type.

Figure 1(a) displays the spectrum for the magnetic energy and k_1 times the helicity spectrum obtained from the Voyager 2 interval. The mean radial solar wind speed for the interval, 442 km/s, has been used to convert from frequency to reduced wavenumber. The behavior shown is fairly typical of near ecliptic fluctuations, see, for example, (6). Figure 1(b) shows the ratio $f = \overline{k}_3/\overline{k}_2$, with the isotropic value of unity overplotted as a dashed line. Clearly, this result is not compatible with isotropy of the fluctuations. Since the interval is characterized by a significant $\mathbf{B}_0$ this lack of isotropy is consistent with expectations based on the dynamical emergence of anisotropy in the presence of mean magnetic fields (10, 17, 20). Nonetheless $\approx 38\%$ of the fluctuations have $\frac{1}{2} < |f| < 2$, so that much of the departure from isotropy is not extreme. The plot also gives an indication of the split between $2\frac{1}{2}$D and slab fluctuations. Pure slab fluctuations are characterized by $f = 0$, whereas the $2\frac{1}{2}$D symmetry has f arbitrary (Table 1). Only about 1% of the $|f|$ values fall below 0.01, which suggests that slab fluctuations are not a dominant component in this interval [cf. Fig. 2(b)].

Turning to the plots of $\overline{k}_2$ and $\overline{k}_3$ (Fig. 2), we observe general increasing trends as k_1 increases, with the scatter about best-fit straight lines being more pronounced for $k_1 \gtrsim 10^{-6}\,\mathrm{km}^{-1}$, which is of order the correlation scale for the magnetic field (see below). The straight lines overplotted on Figure 2(a) are the scalings applicable for fluctuations which are purely $2\frac{1}{2}$D (dashed) and slab (solid) for the same mean field direction as the actual data interval. The agreement at low k_1 between the data and the $2\frac{1}{2}$D prediction is strikingly good, however the interpretation of these scales (larger than the correlation scale) is not unambiguous, *e.g.*, (13). At smaller scales a single pure symmetry state seems an unlikely option. As $2\frac{1}{2}$D modes have arbitrary values for $\overline{k}_3$, it is harder to draw conclusions from Fig. 2(b), although, clearly slab modes are not a dominant component.

The bulk wavenumbers are calculated to be $\overline{K}_2 \doteq -3.2 \times 10^{-9}\,\mathrm{km}^{-2}$, and $\overline{K}_3 \doteq -4.2 \times 10^{-9}\,\mathrm{km}^{-2}$. Taking the square roots and converting to the associated lengthscales gives, respectively, 1.1×10^5 km and 1.0×10^5 km. These values are substantially smaller than the lengthscales associated with the k_1 and k_1^2 moments of H_m^{red}, both $\sim 10^7$ km, and the magnetic

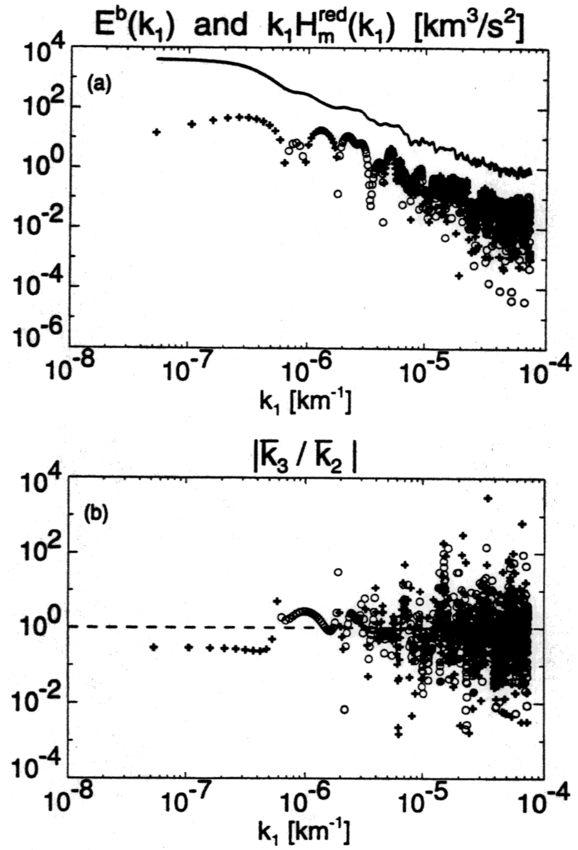

FIGURE 1. Plots of the mean wavenumbers and related quantities for the scalar function $H(\mathbf{k})$, which generates the magnetic helicity, for a Voyager 2 interval near 2.8 AU. Open circles indicate data points with negative values, plus signs those with positive values.

correlation scale $\lambda_b \doteq 5.0 \times 10^6$ km [see (11) for definitions]. This suggests that the magnetic fluctuations contain substantial transverse (helical) structure.

CONCLUSIONS

The technique presented above complements existing methods (2, 9) which help discriminate between various types of fluctuation and may provide further support for the perspective that MHD-scale solar wind fluctuations consist of (at least) two-components: quasi-2D turbulence and slab Alfvén waves (2, 14, 22), or perhaps some other mixture of pure symmetries (3).

We have focussed here on mean wavenumbers associated with the magnetic helicity spectrum. The theory holds, however, for the index antisymmetric part of *any* homogeneous auto or cross-correlation

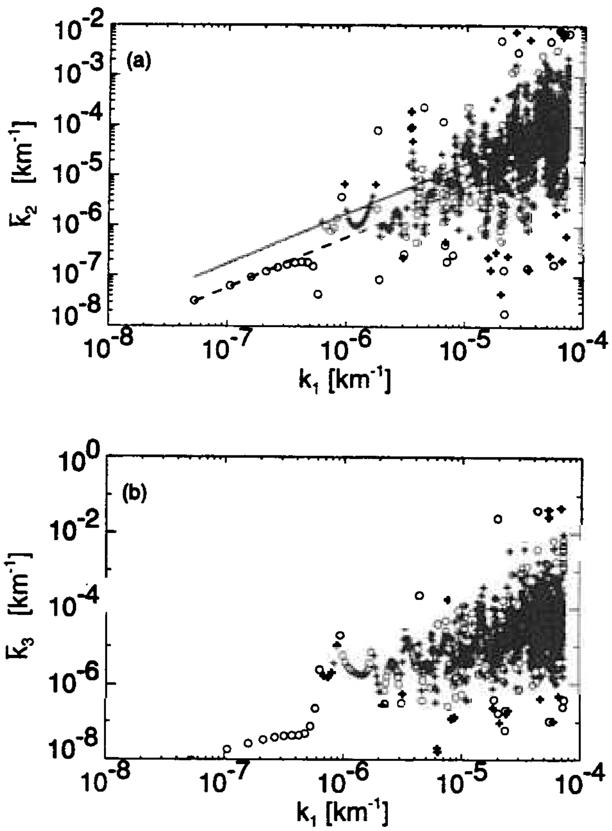

FIGURE 2. Helicity-weighted mean wavenumbers for the 2.8 AU Voyager interval. See Figure 1 (and text).

tensor, when the underlying fields are solenoidal (18). For example, mean wavenumbers can be defined for incompressible velocity fluctuations via the velocity helicity spectrum, and for the induced electric field $\mathcal{E} = -\langle \mathbf{v} \times \mathbf{b} \rangle$ using the antisymmetric piece (J_{ij}^{-}) of the "minus" correlation tensor $R_{ij}^{-}(\mathbf{r}) = \langle v_i b_j' - b_i v_j' \rangle$, where primes indicate evaluation at $\mathbf{x}+\mathbf{r}$ rather than $\mathbf{x}$. As shown in (18), J_{ij}^{-} is directly related to the electric potential for $\mathcal{E}$. On the assumption that solar wind velocity fluctuations are approximately incompressible, one could then obtain $\overline{k}_j$ for these and similar fields and such an investigation is in progress.

Finally, we note that there are many questions still be answered regarding the mean wavenumbers. For example, their variation with heliocentric distance, stream speed, and solar cycle. Data from the Ulysses mission may be particularly useful in this regard as it contains numerous (long) intervals where $\mathbf{B}_0$ is relatively stable.

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