Coronal Heating by Quasi-2D MHD Turbulence driven by non-WKB Wave Reflection

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Abstract. We present a candidate mechanism for heating of the solar corona, via the interaction of (chromospherically-generated) Alfvén waves, their reflections, and quasi-two dimensional (relative to the mean magnetic field) MHD turbulence. The non-propagating nature of the latter means that the restrictive Alfvén timescale constraints associated with high frequency wave heating models are avoided. A phenomenology for this mechanism is described here, and "proof of concept" support from reduced MHD (RMHD) simulations is also discussed. Estimates of the achievable heating efficiency based on turbulence models are favorable, and encourage further investigation of the model's quantitative feasibility.

INTRODUCTION

It seems likely (1, 2) that a physically acceptable model for origin of the high latitude fast solar wind will involve some process(es) which produce significant heat deposition within the first several solar radii above the photosphere, in order to account for the observed rapid acceleration of the wind (3). Thus, the coronal heating and solar wind acceleration problems are tied together. Theory and observations also indicate (4, 5, 6) that the magnetic field plays a crucial role in the heating process, with Alfvén waves and magnetic reconnection as probable agents. Here we present a mechanism for the heating of coronal hole plasma which takes these features into account while avoiding the pitfalls of wave heating models which rely on high frequency waves (4, 7, 8).

The basic physics of the mechanism is shown in Figure 1. The fluctuations are considered to consist of two interacting components: parallel propagating Alfvén (aka "slab") waves and quasi-two dimensional (2D) MHD turbulence. The waves drive the turbulence and the latter dissipates the energy at small perpendicular scales. Specifically, we assume that the continual energy supply for the turbulent heating arises from (approximately) Alfvénic fluctuations, generated in the chromosphere. These propagate up into the corona where some fraction experience non-WKB reflection off the large-scale density and field gradients (9, 10, 11, 12, 13).

When sufficient fluctuation energy resides in a distinct set of modes—namely, low frequency quasi-2D ones, with wavevectors $\mathbf{k}_Q$ almost perpendicular to the (average) coronal magnetic field $\mathbf{B}_0$—then the waves can pump the turbulence (14, 15, 16, 17). At large Reynolds numbers, the quasi-2D fluctuations engage in a transverse turbulent cascade involving successive reconnection of poloidal flux structures, thereby transferring energy to small (perpendicular) scales where it is dissipated. Related studies using solar wind parameters suggest that excitations with highly oblique wavevector can damp due to both ion-cyclotron damping and other processes, such as Landau damping (18, 19).

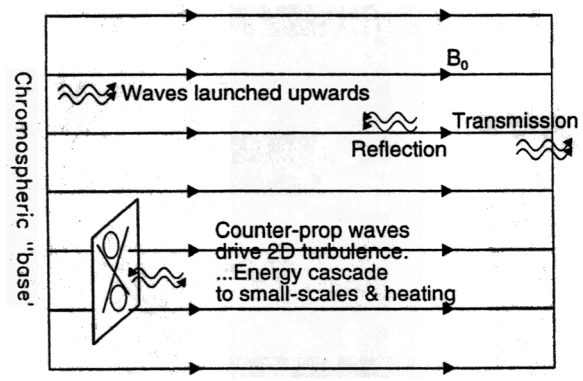

FIGURE 1. Cartoon sketch of the basic physics underlying the proposed coronal heating mechanism.

Several aspects of the model warrant comment. Motivation is dually provided by observations, where

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significant fractions of the fluctuation energy are inferred to be quasi-2D (20), and by results from theory (21, 22, 23) and simulations (14, 15, 16, 24, 17). The latter show that plasmas with a strong B_0 and/or $\beta \ll 1$, such as the chromosphere and corona, dynamically favor development of states dominated by quasi-2D and slab-like modes.

Slab modes, of course, propagate at the Alfvén velocity $\mathbf{V}_A$, so that in the absence of reflections or couplings, their energy would be rapidly transported through the corona. Modes with short enough wavelengths will undergo WKB reflections; however, in the corona such waves are high-frequency and it is unclear whether there is sufficient energy in this frequency band to provide the requisite heating (4, 7). Fortunately, lower frequency modes can undergo non-WKB "mixing" reflections (25) off the gradients in the mean fields. In the corona such gradients are inferred to be strong so that significant non-WKB reflection is likely to occur.

The non-propagating nature of the quasi-2D modes is important: for these modes the associated Alfvén frequency $\omega = \mathbf{k}_Q \cdot \mathbf{V}_A \approx 0$, so that wave effects are of secondary importance. Thus the energy of this component is transported outwards at essentially the flow speed U, which is much less than V_A in this height range. As a consequence, heating resulting from dissipation of the quasi-2D modes occurs more or less (spatially) in place, and at about the right height to fit with observations.

In addition, quasi-2D fluctuations are only weakly affected by B_0, and thus the heating rate for these modes is also essentially independent of the strength of B_0. This may help explain the approximate constancy observed for quantities like the solar wind mass flux and coronal temperature. Details aside, the main tradeoff in the present model relative to models relying upon high frequency wave damping (7) is the following. For damping of high frequency, parallel propagating waves, the cascade in the parallel direction in wavenumber is exceedingly slow (8) and direct cyclotron resonance absorption occurs for progressively lower frequency since the cyclotron frequency decreases with altitude. The present model can begin with waves having essentially the same origin as envisioned in the above models. However, in view of the rapid transverse cascade, we suggest that the restriction on having power in high frequencies is relaxed. In fact, because reflection is expected to be more efficient at lower frequency, the mechanism we propose should actually work better for low frequency wave input. This may be an advantage if low frequency fluctuations are favored in the generation mechanism, as we suspect is the case.

MODEL AND RESULTS

Transport Equations for the Two-Component Model

With the above assumptions it is straightforward to construct a two-component ("slab + 2D") transport model for the coronal (hole) fluctuations, drawing on related models for solar wind fluctuations (26, 27, 28) and phenomenologies for homogeneous MHD turbulence (29). We assume that the fluctuations are incompressible here, although this can be relaxed to near incompressibility (21) without undue hardship. Denote the energy of the upward (downward) high frequency slab waves by w_-^2 (w_+^2). Similarly, let the energy in the quasi-2D low frequency fluctuations be $z_\mp^2$, with associated energy-containing length $\lambda_\perp$. In place of an equation for $\lambda_\perp$ we work with one for $L \equiv z^2\lambda_\perp$, where $z^2 = (z_+^2 + z_-^2)/2$. Collecting the (inhomogeneous) transport terms on the left and the turbulence phenomenology terms on the right, we obtain

$$L^\pm w_\mp^2 = -\frac{w_- w_+ z_*}{\lambda^\perp}, \quad (1)$$

$$L^\pm z_\mp^2 \pm MD = -\alpha\frac{z_\mp^2 z_\pm}{\lambda^\perp}, \quad (2)$$

$$\frac{\partial L}{\partial t} + \mathbf{U}\cdot\nabla L + (\nabla\cdot\frac{\mathbf{U}}{2})(1-\frac{D}{z^2})L = (\alpha-\beta)z_*^3, \quad (3)$$

$$L^\pm = \frac{\partial}{\partial t} \pm \mathbf{V}_A\cdot\nabla \mp (\nabla\cdot\mathbf{V}_A), \quad (4)$$

where $z^* = (z_+^2 z_- + z_-^2 z_+)/(z_+^2 + z_-^2)$ is an average Elsässer velocity, the effective eddy-turnover time is $\tau = \lambda_\perp/z^*$, and the linear $L^\pm$ operators represent WKB effects in the limit $V_A \gg U$. Non-WKB effects appear in connection with the "mixing" operator $M = \nabla\cdot\mathbf{V}_A + \frac{\hat{\mathbf{b}}_i\hat{\mathbf{b}}_j}{\sqrt{4\pi\rho}}\nabla_i B_{0j}$. As the density and mean field vary strongly in this region of the corona, mixing effects are expected to be strong (25). There are no mixing terms in the slab equations since such waves have an energy difference of zero. Typically the MHD Taylor–Karman constants take values $\alpha = 1$ and $\beta = 0$ to 1 (29, 27).

Closure of the model requires an equation for $D = \langle \mathbf{z}_+ \cdot \mathbf{z}_- \rangle$, the energy difference of the quasi-2D fluctuations. One appropriate to coronal conditions is under development; however, motivated by numerous simulation results, here we use the approximation $D/z^2 = \text{const} < 0$.

Analytic solutions to Eqs. (1–4) can be obtained when the RHSs are neglected and simple forms chosen for ρ, U, and $\mathbf{B}_0$ (*e.g.*, powerlaws). For more realistic profiles of the mean fields (30, 31) numerical

solutions will probably be required, perhaps supplemented by asymptotic analysis of the equations.

Note that the models discussed above are of an energy-containing type, *i.e.*, spectral information has been integrated out. Each component is characterized by its relevant Elsässer energies, along with, potentially, a lengthscale for each "energy," although here we describe the model in its simplest incarnation employing just the single characteristic length $\lambda_\perp$. The reflection and transmission coefficients, however, include implicit dependence on various lengthscales.

Homogeneous Two-Component Equations

While the above is perhaps the simplest "waves plus 2D turbulence" coronal heating model, it is nonetheless still quite complicated. We therefore introduce a related model which neglects most transport effects while still including the pivotal ingredients of reflection, transmission, and turbulent decay. Instead of considering the entire system indicated in Figure 1 we consider a sub-box of (homogeneous) coronal material, with boundary and initial conditions chosen to emulate the behavior of a similar box in the larger system.

Constant coefficients are used to model the supply rate of wave energy (F), and the fractions of transmitted (T) and reflected (R_-) upward waves, and reflected downward waves (R_+). The reflection and transmission terms are interpretable as proxies for mixing effects. This leads to a closed model, for now including only the quasi-2D components, consisting of just three equations,

$$\frac{dz_-^2}{dt} = -\alpha\frac{z_-^2 z_+}{\lambda_+^\perp} + F - R_- z_-^2 + R_+ z_+^2 - T z_-^2, \quad (5)$$

$$\frac{dz_+^2}{dt} = -\alpha\frac{z_+^2 z_-}{\lambda_+^\perp} + R_- z_-^2 - R_+ z_+^2, \quad (6)$$

$$\frac{d\lambda^\perp}{dt} = -\beta\frac{\lambda^\perp}{z^2}\left[\frac{dz^2}{dt}\right]. \quad (7)$$

Similar equations for the slab energies can also be written down. Ignoring the (nonlinear) terms modeling turbulent transfer, the equations are linear ODEs with simply obtained analytic solutions. Numerical solution of the full nonlinear ODEs reveals that the system evolves towards a steady-state where the energy leaving it is approximately equipartitioned between transmission and quasi-2D turbulent dissipation, for a wide range of parameters (manuscript in preparation). In other words, the driven system attempts to direct a substantial fraction of its energy into heat.

Connection with RMHD. It is also possible to perform more sophisticated (spectral) simulations of the homogeneous model based on the reduced MHD (RMHD) equations (32, 33, 21). This approximation to the strong B_0 limit of the incompressible MHD equations can be interpreted as planes of 2D turbulence coupled together weakly by long wavelength Alfvén waves. The nonlinear terms are no longer modeled but calculated in full, with low-frequency outward "pump" modes being forced, reflected, and removed ($\sim$ transmitted). Early results from these simulations indicate that driving an initially pure 2D state with just a single upward wave can lead to significant and sustained heating. Thus in the RMHD framework wave input and reflection is very capable of driving roughly steady quasi-2D MHD turbulence. These results indicate heating efficiencies (heating rate/wave energy supply rate) that are comparable to the order-one efficiencies seen in the ODE-based phenomenological model. The RMHD test is more stringent than the ODE-based test. It is also interesting that the initial 2D turbulence stays essentially 2D. There is some transfer of energy to quasi-2D modes, but, perhaps surprisingly, not a great deal to the fully 3D modes (which RMHD supports). The 2D component acts as a catalyst [see (17, 24, 16)] that allows the interaction of upward and downward propagating waves to efficiently (*i.e.*, resonantly) dissipate at high perpendicular wave number. The RMHD simulations show that higher-order (nonresonant) couplings are adequate to maintain this essential catalytic function. The process is essentially nonlinear and crucially dependent upon mode couplings of several varieties (16). Nevertheless it seems to be robust.

CONCLUSIONS

The various numerical results noted above provide a "proof of concept" for the two-component (slab + 2D) coronal heating mechanism. Upwardly launched Alfvén waves can scatter (without satisfying WKB restrictions) and then interact to drive and sustain the initial seed of quasi-2D MHD turbulence. The turbulence undergoes magnetic reconnection and a cascade of energy to small (transverse) lengthscales, where dissipation and concomitant heating proceeds. For the test cases considered so far, it appears to be possible to achieve 50% or better efficiency, *i.e.*, more than half the injected wave energy is deposited

in the (model) corona as heat. It remains to be seen whether or not the quantitative features are consistent with observational constraints.

A crucial feature of the model is the essentially non-propagating nature of the quasi-2D modes (compared to the fast Alfvén speed). Thus, for these fluctuations, the conversion of energy into heat takes place at roughly the same height as the injection of turbulent energy does (via resonant couplings with slab waves). The plentiful Alfvén wave energy is thus damped *indirectly*, by coupling to quasi-2D turbulence and subsequent energy cascade to (small) transverse scales. Moreover, this occurs at a rate independent of the mean field strength and over a height range consistent with observational data. We believe the mechanism is a promising candidate for explaining some parts of the coronal heating problem, with current and future work hopefully clarifying its role and importance.

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