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ANISOTROPY AND ENERGY DECAY IN MAGNETOHYDRODYNAMIC TURBULENCE: THEORY AND SOLAR WIND OBSERVATIONS

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Anisotropies in the spectra of magnetohydrodynamic (MHD) scale fluctuations are observed or inferred in many plasmas, *e.g.,* laboratory fusion machines, the solar wind and corona, and the interstellar medium (see [1, 2] for references). Anisotropy is expected since a mean magnetic field $\mathbf{B}_0$ (unlike a mean flow), cannot be transformed away and thus provides a preferred direction. Understanding the evolution and development of this spectral anisotropy is a major theoretical challenge as nonlinear effects play a crucial role. Nonetheless, some progress has been made. Using numerical simulations and a *reduced* MHD (RMHD) approach, we have developed a relatively simple model of the process [1, 2], which we present below.

Simulations for freely-decaying 2D and 3D incompressible MHD turbulence [1, 2, 3, 4, 5] have shown that for $\delta B/B_0 \lesssim \frac{1}{2}$, spectral transfer in the parallel direction is essentially non-existent, whereas in the perpendicular direction the energy cascade continues out to the dissipation scale in the usual way (directions are relative to $\mathbf{B}_0$, and δB is the rms value of the fluctuating magnetic field). This can be understood in terms of resonant triad interactions involving the Fourier modes. The (Fourier) eigenmodes are left or right propagating Alfvén waves—except that excitations with wavevector $\mathbf{k}$ perpendicular to $\mathbf{B}_0$ are not eigenmodes in the usual sense, although the Alfvén waves can still couple with them. These latter modes are known as zero-frequency or non-propagating modes and can be interpreted as 2D turbulence (with the planes perpendicular to $\mathbf{B}_0$). Calculation of the leading-order nonlinear corrections to the evolution reveals that two modes, $\mathbf{k}_1, \mathbf{k}_2$, will resonantly pump a third, $\mathbf{k}_3$, only if one of the driving

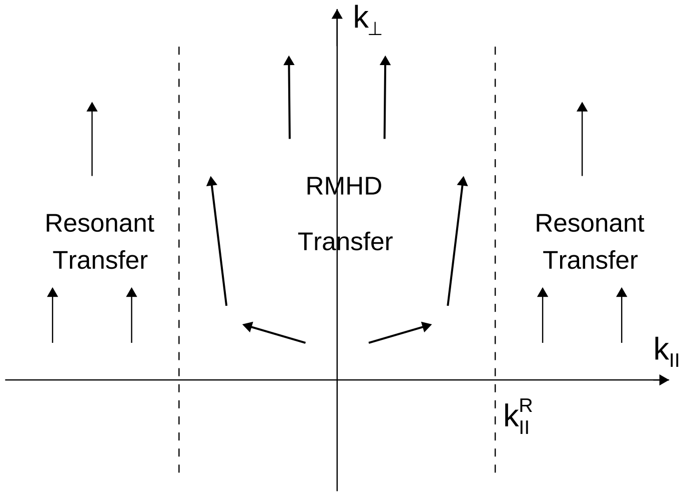

Figure 1. Schematic diagram of the distinct dynamical regions in Fourier space associated with RMHD and resonant triad dynamics.

modes is a zero-frequency one [3]. Since, also, $\mathbf{k}_3 = \mathbf{k}_1 + \mathbf{k}_2$, it is possible to excite modes with higher $k_\perp$, but not higher $k_\parallel$. Thus, there is preferential transfer in the perpendicular directions. Clearly, the presence of a 2D component of the turbulence is crucial to anisotropy development via this mechanism. Despite recent suggestions [6], there appear to be no good *a priori* reasons to discard these components, and moreover caution should be exercised when drawing conclusions about full MHD from analyses based on RMHD [5, 7, 8, 9].

Resonant couplings are not the only way spectral anisotropy is produced, however. A strong $\mathbf{B}_0$ is associated with RMHD processes [10, 11, 5]. RMHD modes are those for which the Alfvén timescale exceeds the large-scale eddy-turnover time, *i.e.,* $1/|\mathbf{k}\cdot\mathbf{B}_0| \gtrsim 1/(k_c \delta B) \Rightarrow k_\parallel \lesssim k_c \delta B/B_0 = k_\parallel^R$, where k_c is the correlation scale for the turbulence. This inequality estimates the minimum parallel lengthscale dynamically excited by RMHD turbulence. For those RMHD modes which qualify, resonant interactions still occur. In addition, however, *all* RMHD modes engage in nonlinear interactions of the familiar Kolmogorov cascade kind, since B_0 (*e.g.,* wave effects) is relatively unimportant for these modes. Thus, excitations cascade out to the dissipation scale in the $\perp$ direction, but only to $k_\parallel^R$ parallel to $\mathbf{B}_0$. It follows that if the initial data contain excitations on both sides of $k_\parallel^R$, there will be two distinct regions in k-space, each governed by a different dynamics (Fig. 1). Modes with $|k_\parallel| \gtrsim k_\parallel^R$ are strongly influenced by Alfvén wave effects, with spectral transfer occuring mainly via the resonant process summarised above. In contrast, the RMHD modes behave more or less like standard MHD turbulence without a mean field, and attempt to maintain roughly isotropic transfer. However, they are stymied in their attempts at this in the parallel direction because of the dynamically imposed RMHD "wall" at $k_{\|}^{R}$. RMHD dynamics does not allow substantial spectral transfer past $k_{\|}^{R}$, since assumptions in the RMHD derivation [10] amount to the condition that $k_{\|} \ll k_{\perp}$ always. Thus, for the RMHD modes also, perpendicular spectral transfer is enhanced.

Using the above arguments one obtains an equation [1, 2] which predicts the degree of anisotropy as a function of $\delta B/B$ and initial condition parameters for $\frac{1}{10} \lesssim \frac{\delta B}{B} < 1$, namely $\cos^2\theta = m(\delta B/B)^2 + c$, where B is the total field magnitude, m and c depend on the turbulence parameters, and θ is the angle between a mean wavenumber and $\mathbf{B}_0$. This linear scaling with $(\delta B/B)^2$ is supported by simulation results for a wide range of flows, including compressible, incompressible, decaying, and driven systems [1, 2].

Spectral anisotropy can also be produced in other ways. When B_0 is strong, for example, the viscosity is no longer isotropic, so that even *linear* dynamics can lead to pronounced anisotropy. Some consequences of this have been examined elsewhere [12, 13].

We anticipate that the above model for the development of spectral anisotropy in MHD turbulence will be useful in understanding the behaviour of many astrophysical and space physics systems. In particular, observations of the solar wind at length-scales of 10^3–10^7 km indicate that the MHD turbulence displays both spectral and variance anisotropy, [14, 15, 16]. Observations also indicate that, usually, $\delta B/B_0 \sim 1$, so that the solar wind plasma is often in the interesting scaling regime. Cosmic ray measurements and theory [17, 18] also suggest that the solar wind consists of two coupled components, namely 2D turbulence and Alfvén waves. The former has been observed [18] to account for as much as 80% of the fluctuation energy, consistent with dynamic evolution of the plasma towards a quasi-2D state as a consequence of anisotropic spectral transfer.

Anisotropy is likely to influence the solar wind plasma in many ways, including spatial transport of turbulence, cosmic ray scattering, and turbulent heating. An example is a simple transport theory [19], employing a local Taylor–Karman decay phenomenology appropriate for quasi-2D MHD, that accounts reasonably well for the radial distribution of turbulent energy from 1 to 40 AU in the low-latitude solar wind. Nevertheless, as noted above, observations seem also to require an additional ingredient, conveniently identified with nearly parallel propagating Alfvén waves. We suggest that a two component dynamical model of energy decay and spectral transfer [14], constrained by decay rates [20] and anisotropy scalings [1, 2] from numerical simulations, may serve as a useful improvement to existing phenomenologies for solar wind and coronal physics problems.

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References

1. Oughton, S., Ghosh, S. & Matthaeus, W. H. (1997) Scaling of spectral anisotropy with magnetic field strength in decaying MHD turbulence. *Phys. Plasmas*, in press.
2. Matthaeus, W. H., Oughton, S., Ghosh, S. & Hossain, M. (1998) Scaling of anisotropy in hydromagnetic turbulence. *Phys. Rev. Lett.*, submitted.
3. Shebalin, J. V., Matthaeus, W. H. & Montgomery, D. (1983) Anisotropy in MHD turbulence due to a mean magnetic field. *J. Plasma Phys.* **29**, 525.
4. Oughton, S., Priest, E. R. & Matthaeus, W. H. (1994) The influence of a mean magnetic field on three-dimensional MHD turbulence. *J. Fluid Mech.* **280**, 95.
5. Kinney, R. & McWilliams, J. C. (1998) Turbulent cascades in anisotropic magnetohydrodynamics. *Phys. Rev. E*, in press.
6. Sridhar, S. & Goldreich, P. (1994) Toward a theory of interstellar turbulence: I. Weak Alfvénic turbulence. *Astrophys. J.* **432**, 612.
7. Montgomery, D. C. & Matthaeus, W. H. (1995) Anisotropic modal energy transfer in interstellar turbulence. *Astrophys. J.* **447**, 706.
8. Ng, C. S. & Bhattacharjee, A. (1996) Interaction of shear-Alfvén wave packets: Implications for weak magnetohydrodynamic turbulence in astrophysical plasmas. *Astrophys. J.* **465**, 845.
9. Goldreich, P. & Sridhar, S. (1997) Magnetohydrodynamic turbulence revisited. *Astrophys. J.* **485**, 680.
10. Montgomery, D. C. (1982) Major disruption, inverse cascades, and the Strauss equations. *Physica Scripta* **T2/1**, 83.
11. Kinney, R. & McWilliams, J. C. (1997) Magnetohydrodynamic equations under anisotropic conditions. *J. Plasma Phys.* **57**, 73.
12. Montgomery, D. C. (1992) Modifications of magnetohydrodynamics as applied to the solar wind. *J. Geophys. Res.* **97**, 4309.
13. Oughton, S. (1996) Ion parallel viscosity and anisotropy in MHD turbulence. *J. Plasma Phys.* **56**, 641.
14. Tu, C.-Y. & Marsch, E. (1995) MHD structures, waves and turbulence in the solar wind. *Space Sci. Rev.* **73**, 1.
15. Matthaeus, W. H., Bieber, J. W. & Zank, G. P. (1995) Unquiet on any front: Anisotropic turbulence in the solar wind. *Rev. Geophys. Supp.* **33**, 609.
16. Matthaeus, W. H., Ghosh, S., Oughton, S. & Roberts, D. A. (1996) Anisotropic three-dimensional MHD turbulence. *J. Geophys. Res.* **101**, 7619.
17. Bieber, J. W., Matthaeus, W. H., Smith, C. W., Wanner, W., Kallenrode, M. & Wibberenz, G. (1994) Proton and electron mean free paths: The Palmer consensus revisited. *Astrophys. J.* **420**, 294.
18. Bieber, J. W., Wanner, W. & Matthaeus, W. H. (1996) Dominant two-dimensional solar wind turbulence with implications for cosmic ray transport. *J. Geophys. Res.* **101**, 2511.
19. Zank, G. P., Matthaeus, W. H. & Smith, C. W. (1996) Evolution of turbulent magnetic fluctuation power with heliocentric distance. *J. Geophys. Res.* **101**, 17 093.
20. Hossain, M., Gray, P. C., Pontius Jr., D. H., Matthaeus, W. H. & Oughton, S. (1995) Phenomenology for the decay of energy-containing eddies in homogeneous MHD turbulence. *Phys. Fluids* **7**, 2886.