1. Verify the formulas A and B given in the lecture notes for the elliptic curve group operation by deriving them using coordinate geometry and a bit of calculus. Then write programs in Mathematica to compute these formulas.

2. Check the values of \((x_n, y_n)\) for \(n \in \{1, 2, 3\}\) for the elliptic curve

\[y^2 = x^3 - 7x + 10\]

given in the lecture notes by showing that they are on the given curve.

3. Consider the elliptic curve \(y^2 = x^3 + 1\). Sketch the curve using the Mathematica function ContourPlot (see the lecture notes for a method). Find its discriminants \(\Delta\) and \(D\). Verify \(P=(2,3)\) generates all of the torsion points of which there are 6. Annotate your sketch with these points and verify they form a group by producing it’s multiplication table. Draw a few lines to show how the group operation works.

4. Note that 7 is a congruent number, but no example of a triangle with area 7 has been given. Use Mathematica to search for \(X < Y < Z\) all in \(\mathbb{Q}\), so that

\[7 = \frac{1}{2}X \cdot Y\]

by searching for an \(x\) with \(x, x-7\) and \(x+7\), all squares of rational numbers. 

[Hint: \(\text{SquareQ}[n] := n == \text{Floor[Sqrt[n]]}^2\) will check to see if a natural number \(n\) is a square.]

5. (Optional) Prove that 1 is not a congruent number.

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