MATH 101A
Tuesday 05 May, 2009.

Test 2

Answer ALL questions. Show ALL working.

Time Allowed: 1.5 hours

No Eton tables. No Calculators.

Section A (40%)

4 **1.** Let $f(x) = 3x^n + 1$, where n = constant. Evaluate $f(f(x) + h)$. $= f(3x^n + 1 + h)$ $= 3[3x^n + 1 + h]^n + 1$

5 **2.** Differentiate $f(x) = e^x + \log(x) + \frac{1}{x^2} + x^{2/3}$. $\Rightarrow f'(x) = e^x + \frac{1}{x} - \frac{2}{x^3} + \frac{2}{3}x^{-1/3}$

5 **3.** Differentiate $F(t) = \frac{\sin(t)}{1+t^2}$. $\Rightarrow F'(t) = \frac{(1+t^2)\cos(t) - \sin(t)[2t]}{(1+t^2)^2}$

5 **4.** Let $F(\theta) = \theta\cos(\theta)$. Determine the FIRST and SECOND derivatives of $F(\theta)$.

$\Rightarrow F'(\theta) = 1\cdot\cos\theta + \theta[-\sin\theta] = \cos\theta - \theta\sin\theta$

$\Rightarrow F''(\theta) = -\sin\theta - \sin\theta - \theta\cos\theta = -2\sin\theta - \theta\cos\theta$

5 **5.** Differentiate $x^3 - xy^2 = 4$ implicitly, and solve for $y'(x) = \frac{dy}{dx}$.

$3x^2 - \frac{d}{dx}[xy^2] = 0 \Rightarrow 3x^2 - [y^2 + x\cdot 2y\cdot y'] = 0 \Rightarrow y' = \frac{y^2 - 3x^2}{2xy}$

7 **6.** Let $f(t) = e^{\sin(at^3)}$, with a = constant. Find $\frac{df}{dt}$, stating any rules you use. Chain with $u = \sin(at^3) = \sin(x)$, $x = at^3$

$\frac{df}{dt} = \frac{df}{du}\cdot\frac{du}{dx}\cdot\frac{dx}{dt} = e^u\cdot\cos(x)\cdot(3at^2) = \ldots$

4 **7.** Evaluate

$$\lim_{x\to 2}\frac{x^2+3x-10}{x-2}. = \lim_{x\to 2}\frac{\cancel{(x-2)}(x+5)}{\cancel{(x-2)}} = 7$$

5 **8.** Evaluate

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}. = \lim_{x\to 9}\left[\frac{\sqrt{x}-3}{x-9}\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right)\right] = \lim_{x\to 9}\frac{(\sqrt{x})^2-3^2}{(x-9)(\sqrt{x}+3)}$$

$$= \lim_{x\to 9}\frac{1}{\sqrt{x}+3} = \frac{1}{3+3} = \frac{1}{6}$$

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eval @ $x=0$ — Calc/a test 2 2004

2 | 1. (a) $f(x) = e^{ax}$ — 1

$f' = a e^{ax}$ — a

$f'' = a^2 e^{ax}$ — a^2

$f^{(n)} = a^n e^{ax}$ — a^n

$\therefore \; e^{ax} = \frac{f(0)}{0!} + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \dots$

$= 1 + \frac{ax}{1!} + \frac{a^2x^2}{2!} + \dots + \frac{(ax)^n}{n!} + \dots$

10 $\quad = \underline{\sum_{n=0}^{\infty} \frac{(ax)^n}{n!}}$

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2 (b) (i) $f''(x) > 0$ on that interval

(ii) $f = x^4 - 4x^3 = x^3(x-4) = 0$ @ $x = 0, 4$

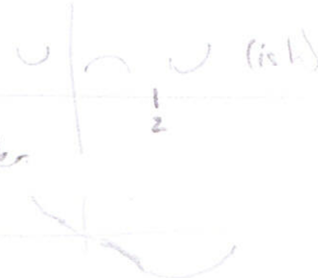
$f' = 4x^3 - 12x^2 = 4x^2(x-3) = 0$ @ $x = 0, 3$

4 tot for f', f'' $\quad f'' = 12x^2 - 24x = 12x(x-2) = 0$ @ $x = 0, 2$

2 $\quad f' < 0$ for $x < 3$ (decr f), $f' > 0$ for $x > 3$ (incr f)

3 $\quad f'' > 0$ for $x < 0$ and $x > 2$: concave up

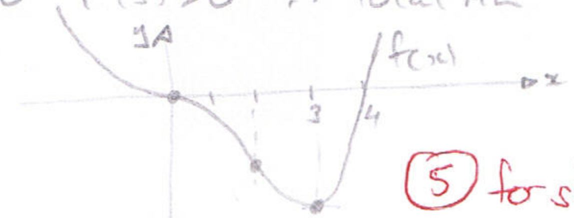
$f'' < 0$ for $0 < x < 2$: down



2 infl. pt. candidates $x = 0$: yes, it is as $f'' > 0$ before and < 0 after

2 ✓ < 0 > 0

2 Max/Min? $x = 3$ $f'(3) = 0$ $f''(3) > 0$ $\therefore$ local min



5 for sketch

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3 1/ a (i) $\frac{dy}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

(ii) $V = \frac{4}{3}\pi r^3(t)$ $\Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

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$\rightarrow 4\pi (10)^2 \cdot 2$

$= 800 \text{ cm}^3/\text{sec}$

2 (b) (i) $f^{-1}(y)$ is the inverse fn for $y = f(x)$

(ii) $y = f(x) = f(x(y))$

$\Rightarrow \frac{dy}{dy} = \frac{df}{dx} \cdot \frac{dx}{dy}$

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$\Rightarrow 1 = \frac{dy}{dx} \cdot \frac{dx}{dy}$ $\Rightarrow \frac{dx}{dy} = 1 / \frac{dy}{dx}$

(iii) $x = \cos^{-1}(y)$

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$\therefore y = \cos(x)$

$y' = -\sin(x) = -\sqrt{\sin^2 x} = -\sqrt{1-\cos^2 x} = \sqrt{1-y^2}$

so $\frac{dx}{dy} = \frac{-1}{\sqrt{1-y^2}}$

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