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Solar Wind Fluctuations: Waves and Turbulence

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Abstract. We present a brief review of observations and theory regarding the nature and radial evolution of MHD-scale solar wind fluctuations. Emphasis is placed on the fact that the fluctuations consist of both waves and turbulence, and on their dual dynamical roles.

1. INTRODUCTION

The realization that the interplanetary medium contains fluctuations in the velocity, magnetic field, and density followed hard on the heels of the presentation of the first theoretical model of the solar wind [1]. Spacecraft observations soon confirmed the existence of such fluctuations, *e.g.*, [2]. Parker's original model [1] predicted flow which was radial with a speed of $\sim 400\,\mathrm{km\,s^{-1}}$ and an Archimedean spiral magnetic field; over forty years later, the *mean* wind is still adequately described and understood using essentially this model. However, understanding of how the associated fluctuations evolve, as they are advected outwards by the mean wind, is much less advanced. Here we review some of the advances in understanding which have occured over these four decades regarding the *origin*, *nature*, and *radial evolution/transport* of the fluctuations. Attention is focused on the role of waves and turbulence in regions close to the ecliptic plane. We denote the fluctuating fields as $\mathbf{v}$, $\mathbf{b}$, and $\delta\rho$, and the mean fields as $\mathbf{U}_0$, $\mathbf{B}_0$, and ρ_0. (Magnetic fields are measured in Alfvén speed units, *e.g.*, $\mathbf{b}$ has been redefined as $\mathbf{b}/\sqrt{4\pi\rho_0}$.)

Table 1 highlights a few important milestones regarding investigations of MHD-scale solar wind fluctuations. Early observations provided evidence for the existence of both Alfvén waves (via highly correlated time series for $\mathbf{v}$ and $\mathbf{b}$, *e.g.*, [3, 4]) and also for strongly nonlinear processes such as turbulence (*e.g.*, power-law energy spectra [5]). However, initial theoretical efforts predominantly assumed that the fluctuations were (Alfvén) waves, and often non-interacting waves, *e.g.*, [3, 6–9], although see [5]. See Table 2 for some important distinctions between the behaviour of waves and turbulence.

Prior to the 1980s the associated transport and evolution models typically employed (leading-order) WKB theory, wherein the wavelength of the fluctuations is assumed much smaller than the lengthscale on which the mean fields vary. While this assumption is well-grounded, it is now recognised that it it is a necessary—but not sufficient—condition for the validity of WKB theory as applied to MHD-scale fluctuations in a supersonic and super-Alfvénic flow [10–12]. Moreover, in standard leading-order WKB theory there is no interaction between inward and outward type fluctuations, where these terms refer to the sense of mode propagation along $\mathbf{B}_0$ (however, see [10–13]).

TABLE 1. A rough (decadal) history of solar wind fluctuation studies

1950s	Birkeland deduces existence of solar wind. Parker proposes a theoretical model for it [1].
1960s	First satellite data; Power-law spectra and evidence for waves. First application of WKB theory to evolution of fluctuations [8]. Suggestion that fluctuations are turbulent [5].
1970s	Plentiful wave observations in ecliptic plane. Many wave transport models (mostly WKB).
1980s	First turbulent transport models and analysis [14–22].
1990s	Newer data (Ulysses, WIND, ACE, ...). Improvements in both transport models and understanding of the nature of fluctuations.

In fact, many of the predictions of WKB theory disagree with observational results (see below). These shortcomings prompted consideration of various other approaches, including turbulence-based transport models.

Before proceeding to the observational results, it is instructive to discuss the major factors which influence the evolution of solar wind fluctuations (Table 3). Although expansion effects dominate the radial evolution behaviour, it is departures from this spherical expansion which comprise the "interesting" physics, since the former holds little mystery. The remaining factors in the table can lead to such departures. All fluctuations are carried along by the mean wind, and since at small enough lengthscales the flow is essentially uniform, fluctuations

TABLE 2. Some distinctions between turbulence and waves

TURBULENCE	WAVES
Inherently nonlinear $\Rightarrow$ *spectral* transfer.	(Small-ampl.) theory = linear $\Rightarrow$ no spectral transfer.
Advection ("self"-distortion).	Propagation $\rightarrow$ *spatial* transport of energy.
No dispersion relation; all dynamical length/ time-scales coupled.	Dispersion relation, *i.e.*, each timescale depends on only a few lengthscales *e.g.*, $\omega = \pm \mathbf{k} \cdot \mathbf{V}_A$

TABLE 3. Some factors influencing fluctuation evolution

Expansion	Dominant factor. Controls overall "decay" with R.
Advection	By mean flow.
Preferred directions	Radial and (local) mean magnetic field. At least partly responsible for various observed anisotropies.
Nonlinear interactions	*Spectral* transfer of energy (from large scales to small ones).
Wave effects	Propagation. *Spatial* transport of energy.
Large-scale gradients	So-called MECS effects (Mixing, Expansion, Compression, Shear): Gradients in mean fields mediate couplings amongst fluctuations.
Pickup ions	Inject energy into fluctuations. Important for $R \gtrsim 8$ AU.

at these scales are unaltered by the advection. On the other hand, larger scale fluctuations notice the spatial dependence of the mean wind and are influenced by the associated gradients. This is one contribution to the MECS effects also listed in the table. Note that these are typically non-WKB effects [10–12, 21, 23–25], which bleed energy from the mean fields into fluctuations, as in the Kelvin–Helmholtz instability. It is straightforward to estimate that the lengthscales on which these large-scale gradients vary is of order the heliocentric distance R [21, 26]. For example, in spherical geometry $(\nabla \cdot \mathbf{U}_0)/U_0 = 2/R$ for $\mathbf{U}_0 = U_0 \hat{\mathbf{R}}$. Nonlinear interactions tend to cause *spectral* redistribution of energy, primarily transfer from large scales to small ones, whereas wave effects are associated with *spatial* transport of energy.

The influence of the two preferred directions (radial and $\mathbf{B}_0$) is also significant. The simplest models of the solar wind are often one-dimensional, with that direction being the radial since this is parallel to the direction of the mean flow. More sophisticated 3D models further emphasize the importance of the radial direction to the fluctuation dynamics, due to both expansion and shear effects; *e.g.*, [27–30]. A strong mean $\mathbf{B}$ tends to induce the dynamical development of spectral (and sometimes also variance) anisotropies, in both incompressible and compressible plasmas, *e.g.*, [31]. The effect is to make the flow quasi-two-dimensional (quasi-2D) in the sense that correlation lengths perpendicular to $\mathbf{B}_0$ become much shorter than those parallel to it, *e.g.*, [32–34].

2. OBSERVATIONAL RESULTS

These have been well-reviewed elsewhere (*e.g.*, [35–37]) so that here we present only a few pertinent points regarding radial evolution and the geometry of the fluctuations.

2.1. Radial Evolution

Observational studies of *in situ* data collected near the ecliptic in both the inner and outer heliosphere suggest that certain features of the radial evolution of fluctuations are rather robust [14–17, 38–43].

The magnetic energy (per unit *mass*) of the fluctuations, denoted $\langle \mathbf{b}^2 \rangle$ with $\langle \cdots \rangle$ thought of as a spatial average (and $\mathbf{b}$ in Alfvén speed units), typically decreases with heliocentric distance and indeed inside ~ 8 AU follows an approximate powerlaw of $\sim R^{-1}$. Recall that (leading-order) WKB theory predicts an R^{-1} dependence [8]. However, one should also note that the observed dependence is not a strict powerlaw, with scatter in the data probably being due to multiple effects including variations in wind speed, solar cycle phase, and distance from the current sheet. For distances $\gtrsim 8$ AU, the magnetic energy is significantly above the WKB powerlaw level and the suggestion has been made that this "excess" energy is provided by pickup ions [24, 44–46].

Kinetic energy per unit mass, $\langle \mathbf{v}^2 \rangle$, often evolves similarly to the magnetic energy, and it is convenient to consider the evolution of their ratio, known as the the Alfvén ratio $r_{\mathrm{A}} = \langle \mathbf{v}^2 \rangle / \langle \mathbf{b}^2 \rangle$. At around 0.3 AU this takes values near to or slightly larger than unity. Thereafter, it tends to decrease with distance, to a value of $\approx \frac{1}{2}$ by 5 AU and remain at roughly this level out to at least 20 AU [47].

Note that r_{A} can be used as a diagnostic of the prevalence of fluctuations which are Alfvén waves. This follows since the energy in an individual Alfvén wave is equipartitioned between its kinetic and magnetic components (when averaged over a wave period) and thus should have $r_{\mathrm{A}} = 1$. Departures from this value suggest the presence of fluctuations which are not Alfvén waves, such as turbulence. In particular, 2D and 3D MHD simulations with an energetically weak $\mathbf{B}_0$ tend to have $r_{\mathrm{A}} < 1$, indicating an excess of fluctuation magnetic energy, *e.g.*, [33, 48–50]. This may be associated with the current sheets and vorticity quadrupoles characteristic of magnetic reconnection sites [50]. Alternatively, $r_{\mathrm{A}} < 1$ may be indicative of local (in Fourier space) dynamo ac-

tion in the turbulent magnetofluid [51]. These two explanations for $r_A < 1$ are based on nonlinear processes. In addition, modeling results indicate that the expansion itself, in the form of "mixing" effects—which are linear (see §3.1)—can also lead to $r_A < 1$ [52, 53].

Another quantity which can provide information regarding the abundance of waves is the normalised cross helicity: $\sigma_c = 2\langle \mathbf{v}\cdot\mathbf{b}\rangle/[\langle \mathbf{v}^2\rangle + \langle \mathbf{b}^2\rangle]$. This is defined such that $|\sigma_c| < 1$, with purely outward propagating Alfvén waves having $\sigma_c = 1$. Helios and Voyager observations found that σ_c decreased from values of ≈ 1 near 0.3 AU, to values scattered around zero beyond about 5 AU [42]. Note that this is very different from the (leading-order) WKB prediction of $\sigma_c \approx 1$ [7]. (Systematic behaviour of σ_c at *high* latitudes has also been studied [40].)

Density fluctuations are typically observed to be $\sim$ 10% at small (1 hour) scales, in both compression and rarefaction regions [43]. Such results provide support for the use of incompressible and nearly incompressible models for the fluctuations [54–56] (however cf. [57]).

Although it is difficult to give a unique interpretation to the behaviour summarised above, one can nonetheless conclude that (i) fluctuations are not just (outward) propagating Alfvén waves, and (ii) inward-type modes tend to become relatively more abundant with distance.

2.2. Fluctuation Geometry

Determining the nature of the geometry (*e.g.*, quasi-2D *vs*. slab waves) of the fluctuations using data from a single spacecraft is problematic for at least two reasons. First, even full (vector) amplitude information is often insufficient to distinguish between different types of fluctuation. For example, both (quasi-)parallel-propagating Alfvén waves and (quasi-)2D turbulence have amplitudes which are (almost) transverse to $\mathbf{B}_0$. Consequently, they both have a minimum variance direction (MVD) which is $\approx \hat{\mathbf{B}}_0$, although their Fourier wavevector orientations are very different (respectively $\parallel$ and $\perp$ to $\mathbf{B}_0$.)

Second, single spacecraft data typically consists of time series at approximately the same spatial position which can be used to calculate power spectra as a function of (Fourier) frequency, $P(f)$, say. Invoking the Taylor frozen-in flow hypothesis (justified because of the supersonic flow speed of the wind) enables these frequency spectra to be converted into *reduced* wavenumber spectra, $S^{\rm red}(k_{\rm red}) = \int S(\mathbf{k})\mathrm{d}\mathbf{k}_\perp$, where $S(\mathbf{k})$ is the full wave*vector* spectrum and integration is over coordinates perpendicular to the sampling one. Unfortunately, except for particularly strong symmetries (*e.g.*, isotropic) determination of fluctuation geometry usually requires knowledge of the full wave*vector* spectrum [58, 59].

Thus, most spacecraft datasets do not support full determination of fluctuation geometry. Nonetheless, using various strategies it is sometimes still possible to extract additional information regarding the nature of the fluctuations. For example, one can analyse data intervals which have $\mathbf{B}_0$ oriented at many different angles to the sampling (radial) direction. Such an analysis has been performed [60], and under the assumption of axisymmetry about $\mathbf{B}_0$, a magnetic correlation function was obtained as a function of coordinates parallel and perpendicular to $\mathbf{B}_0$ (the so-called "Maltese cross"). The results indicate that the fluctuations could consist of two (or more) distinct populations, the first characterised by little variation in the perpendicular directions (*e.g.*, slab waves), and the second by little variation in the parallel direction (*e.g.*, quasi-2D turbulence). Carbone et al. [61] performed a related study with the dataset restricted so that only Alfvénic intervals were used and an assumed decomposition into linear wave modes. They also found evidence for a two-component nature of the fluctuations, although of a different kind to the Maltese cross study, presumably because of the differences in both the data selection policies and in the underlying assumptions in the data analysis. In any case, "two-component" descriptions of solar wind fluctuations have subsequently been widely employed (cf. §3).

More quantitative results have also been presented. Bieber et al. [62] used data for cosmic ray mean free path lengths to infer that the energy partitioning for a quasi-2D/slab model for solar wind fluctuations would be $\approx$ 80%-20%. In a direct test based on observed (inertial range) power spectra, it was also found that this 80-20 partitioning produced the best fit to the data [63]. Still a third test derives from Mach number scalings associated with nearly incompressible theory [54], and for typical observed Mach numbers is also consistent with the 80-20 split. Thus there is abundant—and consistent—evidence for a slab/quasi-2D two-component description, with the quasi-2D component being energetically dominant.

Efforts have also been made to explain the dynamical origin of the two-component model(s) using simulation studies [64, 65], and have met with some limited success. The suggestion is that the two components can appear at different stages of the evolution, or alternatively that the "2D" component is associated with non-zero (but still small) $k_\parallel$ values, and is therefore actually *quasi*-2D.

Returning to the MVD data, Voyager observations inside of 10 AU indicate that the MVD for $\mathbf{b}$ is centered around the local $\mathbf{B}_0$ direction, and similarly for the $\mathbf{v}$ fluctuations, although their MVD moves towards the radial with increasing R [66]. Compressible (polytropic) 3D MHD simulations can reproduce similar MVD data, both in terms of direction and power ratios between components, although there is quite a strong plasma beta de-

pendence [32]. Also contentious is the issue of whether the MVD is aligned with the fluctuations wavevector, as would be the case for Alfvén waves but not for quasi-2D fluctuations [60, 66].

3. TURBULENCE-BASED MODELS

The first published suggestion that solar wind fluctuations could be turbulent appears to be due to Coleman [5]. It took another 16 years, however, for the first turbulence-based transport model for fluctuations to appear [20]. Prior to this, transport models were predominantly based on the application of WKB theory to short-wavelength Alfvén waves [6–9, 67, 68], although there were some studies which explored non-WKB effects, *e.g.*, [69].

Results from WKB theory are, in general, *not* in agreement with observations,[1] with some of the more well-known failures listed in Table 4 [10, 21, 37, 66]. Consequently there was a need to develop non-WKB models.

Below we briefly review a particular class of turbulence-based models which have had some success in matching a wider range of observational data. Note that various other interesting and important non-WKB models have also been investigated, such as the *expanding box model* of Grappin and Velli [27, 28], but they are not our focus here (see also [71–73]).

3.1. Scale-separation models

The underlying idea here is similar to that of the Reynolds decomposition in classical hydrodynamic turbulence theory *e.g.*, [74]. More specifically, one assumes that $\mathbf{V}$, $\mathbf{B}$, and ρ can each be meaningfully decomposed into a mean component and a fluctuating component using a multiple-scales approach. For example, that $\mathbf{V} = \mathbf{U}_0(\mathbf{R}) + \mathbf{v}(\mathbf{x},t;\mathbf{R})$, where $\mathbf{R}$ is the heliocentric position vector, $\mathbf{x}$ is a small spatial displacement at each given $\mathbf{R}$, and $\mathbf{U}_0(\mathbf{R}) = \langle \mathbf{V} \rangle$ can be thought of as the average of the velocity with respect to the small scales, $\mathbf{x}$.

An important point is that whereas in the WKB approach fluctuations are assumed to be Alfvén waves from the outset, the present approach requires no such assumption: the fluctuations can be waves and/or turbulence. Also, the amplitude of the fluctuations need not be small.

Equations for the evolution of the fluctuations can be obtained by substituting these decompositions into the MHD equations, averaging, and then subtracting the results from the unaveraged equations. Switching to Elsässer variables, $\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{b}$, the equations can be written

$$\left(\frac{\partial}{\partial t} + L_{\pm}^{WKB}\right)\mathbf{z}^{\pm} + M^{\pm}\mathbf{z}^{\mp} \quad = \quad NL^{\pm}, \qquad (1)$$

where the (tensor) operators $L_{\pm}^{WKB}$ represent WKB effects and the $M^{\pm}$ "mixing" effects, which depend solely on the gradients of the mean fields. Nonlinear terms are collected on the RHS and denoted only symbolically. Note that the mixing effects are linear and appear at the same formal order as the WKB effects [11, 12, 21, 23].

While it is sometimes advantageous to work directly with these equations [11, 24, 25], it is often more convenient to form the equations for the evolution of the associated spectra (or, equivalently, correlation functions). To make further progress, one can assume that the fluctuations have particular symmetry properties, based on observational [60–63] and theoretical results [75, 76]. In the most general case this would yield 16 transport equations, one for each of the 16 scalar functions (such as energies and helicities) which collectively characterise the fluctuations [21, 76]. In some of the simpler cases the equations reduce to

$$\frac{\partial E^{\pm}(\mathbf{k})}{\partial t} + L_{WKB}^{\pm}E^{\pm}(\mathbf{k}) + M^{\pm}F(\mathbf{k}) = NL^{\pm}(\mathbf{k}) + S^{\pm}, \quad (2)$$

where $E^{\pm}(\mathbf{k}) = \langle \mathbf{z}^{\pm}(-\mathbf{k}) \cdot \mathbf{z}^{\pm}(\mathbf{k}) \rangle$ are the spectra for the Elsässer energies, $F(\mathbf{k}) \sim \langle \mathbf{z}^{+}(-\mathbf{k}) \cdot \mathbf{z}^{-}(\mathbf{k}) \rangle \propto \langle \mathbf{v}^2 - \mathbf{b}^2 \rangle$, and at least two more equations similar to (2) are required to close the system [13, 21, 23, 25, 77]. Also included are possible source terms, $S^{\pm}$. Although the associated models by no means reproduce all the observational trends, the degree of agreement is encouraging [21, 24, 37, 52, 70, 77–80].

Recently, this approach was used to model the radial evolution of $\langle \mathbf{b}^2 \rangle$, a correlation length for the magnetic energy, and the proton temperature T_p, from 1–40 AU [70, 78]. Three major assumptions were adopted, namely that (i) fluctuations are primarily quasi-2D turbulence (with $\sigma_c \approx 0$), (ii) energy is injected into the fluctuations by large-scale shear and compressions, and also, at large R, by pickup ions, and (iii) energy cascades to small *perpendicular* scales where it is converted into heat.

TABLE 4. Some failures of (leading-order) WKB theory

WKB PREDICTION	OBSERVATION ($1 \lesssim R \lesssim 10$ AU)
No inward modes above Alfvén critical radius.	Significant amounts
$r_A \approx 1$; $\sigma_c \approx 1$	$r_A \approx 0.5$; $\sigma_c(R) \rightarrow \approx 0$
MVD for $\mathbf{v}$, $\mathbf{b}$: radial	$\approx$ parallel to $\hat{\mathbf{B}}_0$ (but that for $\mathbf{v}$ moves closer to $\hat{\mathbf{R}}$ with incr R)

[1] The major exception is the radial evolution of $\langle \mathbf{b}^2 \rangle$ inside ~ 10 AU, which is indeed close to the WKB prediction of R^{-1}. However, it turns out that many other approaches also yield this form. See *e.g.*, [24, 47, 70] and discussion below.

Using the scale-decomposition approach described above, one obtains a system of three (ordinary) differential equations, assuming steady-state. The linear structure is of the form (2)—although further simplifications are made—while the nonlinear terms are modeled using homogeneous turbulence theory (*e.g.*, [81] and references therein). Source terms modeling the injection of pickup ion energy are also included and these start to become effective for $R \gtrsim 8\,\mathrm{AU}$. Numerical solutions of the equations for realistic values of the variables at the inner (1 AU) boundary, and the wind speed, etc. show excellent agreement with observational data from the Voyager and Pioneer missions in the case of $\langle \mathbf{b}^2 \rangle$ and T_p (the agreement with the correlation scale is less persuasive).

Richardson has recently improved this model by using the observed correlation between wind speed and T_p at 1 AU. This produces a model wherein the radial temperature dependence fits the observational peaks and troughs quite strikingly (see his paper in this volume).

Finally, we wish to comment on the apparent success of WKB theory in predicting $\langle \mathbf{b}^2 \rangle \sim R^{-1}$ [24, 47, 53, 70]. For reasonable models of the mean fields the mixing terms (*e.g.*, $M^{\pm}F$) are only important for $R \lesssim 2\,\mathrm{AU}$ [21, 52]. Thus, in the outer heliosphere departures from WKB evolution of the energies would appear to be due to nonlinear effects and/or source terms [see Eq. (2)]. However, for inertial range scales in strong turbulence $NL^{\pm}(\mathbf{k}) \approx 0$ [*e.g.*, 21]. Thus, in the absence of sources (2) reduces to the WKB energy transport equation and the WKB prediction is recovered (coincidentally), *despite its lack of applicability* in these circumstances.

4. CONCLUSIONS AND SUMMARY

It is now well-accepted that solar wind fluctuations include both turbulence and waves, with each playing important roles. Models based purely on wave effects generally give inadequate agreement with observations, whereas turbulence-based models are giving encouraging agreement over an enormous range of distances [13, 21, 23–25, 37, 70, 77, 78, 80].

Returning to the three aspects of solar wind fluctuations mentioned in the introduction (origin, evolution, and nature), we note a few points about each one.

Origin. Outward propagating waves probably predominantly originate in the corona, with the bulk of the inward type modes being generated by (spatially local) *in situ* dynamics. Generation of the inward type modes could be due to either linear effects (*e.g.*, mixing) or nonlinear ones, such as turbulence, or a combination.

Evolution. Despite some apparent successes, the WKB approximation is usually inappropriate and inadequate, whereas observations, theory, and modeling all suggest that turbulence is likely to play a crucial role. In particular, driving by shear appears to be essential in accounting for the observed super-adiabatic $T_p(R)$ profile.

Nature. The fluctuations are definitely not pure Alfvén waves (either outward type or a linear superposition of inward and outward), although such intervals may exist. There is multiple support—from observations, theory, and simulations—for the fluctuations being comprised of (at least) two components, namely quasi-parallel-propagating Alfvén waves and quasi-2D turbulence, with the latter energetically dominant.

Finally, we note that there are many other issues pertaining to solar wind fluctuations which we have not discussed here. These include the details of the dissipation mechanism, the difficulty of achieving a parallel cascade of energy in MHD when $\langle \mathbf{b}^2 \rangle / B_0^2 < 1$, the situation out of the ecliptic, the radial evolution of spectra, and variance and polarization anisotropies. Many of these topics are discussed elsewhere in this volume.

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