CORONAL HEATING VIA ALFVÉN WAVES AND 2D MHD TURBULENCE

S. Oughton[1], W.H. Matthaeus[2], G.P. Zank[2], D.J. Mullan[2]
[1]Department of Mathematics, UCL, London WC1E 6BT, UK
[2]Bartol Research Institute, University of Delaware, Newark DE 19716, USA

ABSTRACT

A new model for the heating of coronal hole plasma is presented. The model is based on the (non-WKB) reflection, off gradients in the mean fields, of upward travelling Alfvénic disturbances. The reflection leads to a population of counter-propagating modes which interact with quasi-2D MHD turbulence to drive the latter. This in turn experiences a cascade of energy to small *perpendicular* length-scales (relative to $\mathbf{B}_0$) where dissipation occurs, along with the concomitant deposition of heat. Promising advantages to the model are discussed and "proof of principle" support from reduced MHD simulations presented. Estimates of achievable heating efficiencies based on the simulations are consistent with phenomenological treatments of the same model, and encourage further investigation of the model's quantitative feasibility.

Key words: coronal heating; MHD turbulence; Alfvén waves.

1. INTRODUCTION

Observational and theoretical evidence for the acceleration of the high latitude fast solar wind indicates—one might almost say requires—that there is significant deposition of heat within the first few solar radii (R_s) above the photosphere (Habbal et al. 1995; McKenzie et al. 1995; Grall et al. 1996), thereby linking the coronal heating and solar wind acceleration problems. Moreover, the heating process is likely to rely heavily on the coronal magnetic field (Axford & McKenzie 1997; Culhane 1998; Ulmschneider et al. 1991), with Alfvén waves and magnetic reconnection probably playing leading roles. While much effort has been expended in attempting to explain the origin of the high coronal temperature, no completely satisfactory explanation has emerged as yet although significant progress has been made. For example, a promising model based on the *direct* (cyclotron) damping of high frequency (~ 0.1–$10\,$kHz) slab Alfvén waves has been proposed recently (McKenzie et al. 1995; Axford & McKenzie 1997).

In this paper we present a two-component (low frequency waves plus 2D turbulence) model for heating of coronal hole plasma which is related, but nonetheless distinct from the Axford–McKenzie one. The crucial difference is that it is based on the *indirect* damping of *low* frequency Alfvén waves, with the wave energy being channeled first into quasi-two dimensional MHD turbulence, then to small scales, and finally into heat. As will be seen, this avoids some of the difficulties associated with high-frequency wave-heating scenarios. In particular, there is no need to posit an anomalously high level of (unobserved) wave power at high frequencies (McKenzie et al. 1995; Axford & McKenzie 1997; Tu & Marsch 1997).

There appear to be excellent grounds for pursuing a low frequency wave model of this type and several lines of investigation are being considered. These include phenomenologies for the wave–turbulence coupling in a typical parcel of coronal plasma (Matthaeus et al. 1999), transport model(s), perhaps derived from multiple-scales analysis of the MHD equations, which incorporate appropriate boundary conditions, and, finally, simulations of the full nonlinear equations, for either a typical plasma parcel again, or for the full geometry. Here, we focus on results from reduced MHD (RMHD) simulations of a typical portion of the corona. The phenomenological treatment of the model (Matthaeus et al. 1999) is complementary to the present treatment. In that case, the turbulence is assumed to be fully developed and is treated using a one-point phenomenological closure. Whereas in this paper the turbulence, although represented in an idealized geometrical setting, is treated explicitly, including all relevant nonlinear couplings that generate the cascade responsible for heating.

2. MODEL

The basic physical picture associated with the heating model is as follows (figure 1). Alfvénic fluctuations, generated in the photosphere and/or chromosphere, are launched up into an open field-line region of the corona. While propagating upwards they experience (non-WKB) reflections off the gradients in the background density and magnetic fields (Zhou & Matthaeus 1990b; Velli 1993). This yields a population of counter-propagating waves which couple nonlinearly with quasi-2D modes. The (driven) quasi-2D dynamics is such that energy cascades to small perpendicular lengthscales (relative to $\mathbf{B}_0$), where it is

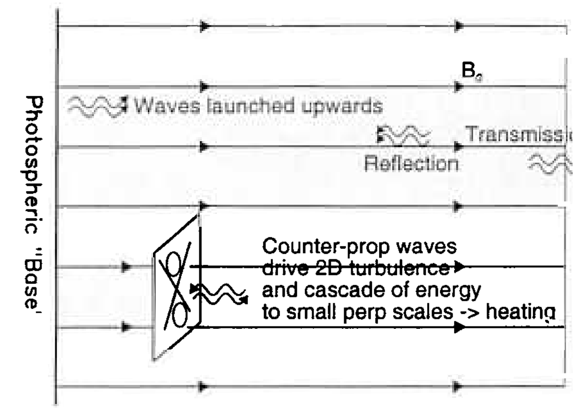

Figure 1. Cartoon sketch of the physics associated with the model for heating coronal hole plasma.

deposited as heat. The energy cascade is likely to involve the formation of current sheets and the (successive) reconnection of magnetic islands (*e.g.,* Hossain et al. 1985; Matthaeus & Lamkin 1986). As indicated in figure 1, allowance is also made for waves propagating out of the "top" of the heating region.

A Fourier mode, $\mathbf{v}(\mathbf{k})$ say, is defined to be quasi-2D if both $\mathbf{k} \cdot \mathbf{B}_0 \approx 0$ and $\mathbf{v}(\mathbf{k}) \cdot \mathbf{B}_0 \approx 0$, where $\mathbf{B}_0$ is the mean magnetic field. While here we are interested in the non-linear dynamics of such modes, it is worth emphasizing that even in the linear limit quasi-2D modes are not, in general, waves in the usual sense. This is because fluctuations which are purely 2D ($\mathbf{k} \cdot \mathbf{B}_0 = 0$) are strictly non-propagating. In addition (see below) waves with non-zero but very low frequency are significantly perturbed by nonlinear couplings.

There are at least four important advantages to this mechanism which heating models based on direct wave-damping can lack. First, since the heating occurs via *indirect* damping of the driving Alfvén waves (upwards and downwards), the restrictive constraints associated with "box-crossing" timescales play no important role. All that is required is that enough energy be bled out of the propagating wave population to sustain the quasi-2D turbulence. As waves are constantly being generated, at the coronal base and by reflection, the reservoir of energy to drive the turbulence is maintained.

In this context, it is worth recalling that when dissipation is achieved through a (statistically steady) turbulent cascade of energy, the dissipation rate is set by the energy injection rate. The energy cascade provides, so to speak, a pipeline between the large energy injection scales and the small dissipation scales. For roughly steady driving of the turbulence the timescale for transit of the pipeline is the large-scale eddy-turnover time $\tau_{\rm NL} = \lambda_\perp/Z$, where Z^2 is the energy in the turbulence fluctuations and $\lambda_\perp$ is a characteristic lengthscale for the, in this case, quasi-2D turbulence. Note that this is independent of the Reynolds numbers (*cf.* Chiuderi et al., this volume), and also of the periods of the driving waves, except in so far as the nonlinear interaction time for the counter-propagating waves is period-dependent.

Second, the plasma heating occurs essentially "in place", that is, at much the same height at which the wave energy is injected into the quasi-2D component. Two factors contribute here: (i) the speed of the nascent solar wind is inferred to be low in the region where the heating process is most relevant (Grall et al. 1996; Guhathakurta & Sittler 1999; Sittler & Guhathakurta 1999), ensuring that advection does not move the energy very far outwards spatially as it undergoes spectral transfer to small perpendicular scales; and (ii) although the quasi-2D fluctuations have long parallel ($\approx$ vertical) lengthscales, the energy transfer to them is initiated at the heights where the upward and downward modes spatially overlap. While this region is relatively extended, given that factor (i) holds, the cascade and heating will still take place in approximately the same height range.

A third advantage is that the dynamics of quasi-2D turbulence is essentially insensitive to the mean field strength, B_0 (*e.g.,* Oughton et al. 1994; Hossain et al. 1995; Matthaeus et al. 1996; Kinney & McWilliams 1998; Matthaeus et al. 1998; Oughton et al. 1998). Moreover, the wave–turbulence coupling strength(s) is also relatively insensitive to B_0, provided that $Z^2/B_0^2 \lesssim 1$. This condition is expected to hold in the corona. Thus, the heating process as a whole depends only weakly on B_0, which is consistent with the observed (approximate) solar cycle independence of quantities like the coronal (hole) temperature, solar wind mass flux, and maximum fast solar wind speed.

Finally, because the non-WKB reflection is more effective for lower-frequency waves (Moore et al. 1991; Musielak et al. 1992; Velli 1993), the present model is most efficient when driven by such waves. Observations indicate that there is adequate power available to heat the corona in this portion of the power spectrum, the problem being how to extract it. On the other hand, evidence for sufficient power at higher frequencies is currently scant (McKenzie et al. 1995; Axford & McKenzie 1997; Tu & Marsch 1997).

3. REDUCED MHD FORMULATION

The equations of RMHD are essentially those of incompressible 2D MHD with allowance made for long-wavelength Alfvén wave-like couplings between the 2D planes (Montgomery 1982; Zank & Matthaeus 1992). The retained dynamics satisfies $\tau \lesssim \tau_{\rm A}$, where τ is the characteristic timescale of an RMHD mode of wavenumber $k = |\mathbf{k}|$, and $\tau_{\rm A} = 1/|\mathbf{k} \cdot \mathbf{B}_0|$ is its Alfvén crossing time. Theory and simulations indicate that in the presence of a sufficiently strong mean field or for sufficiently small plasma beta, 3D MHD turbulence dynamically reorganises to favour RMHD-type fluctuations (Montgomery 1982; Montgomery & Turner 1981; Shebalin et al. 1983; Zank & Matthaeus 1992; Oughton et al. 1994; Kinney & McWilliams 1998; Matthaeus et al. 1998; Oughton et al. 1998). Hence RMHD is usefully employed in low plasma-β systems such as the solar corona. In standard (non-dimensionalized) notation, the RMHD equations for the evolution of the vorticity ω and the magnetic potential a are

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\omega \;=\; \mathbf{b}\cdot\nabla j + \nu\nabla^2\omega + B_0\frac{\partial j}{\partial z}, \quad (1)$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)a \;=\; \eta\nabla^2 a + B_0\frac{\partial \psi}{\partial z}. \quad (2)$$

The equations are closed by noting that $\nabla\cdot\mathbf{v} = 0$, $\omega = -\nabla^2\psi$, and $\mathbf{v} = (\frac{\partial\psi}{\partial y}, -\frac{\partial\psi}{\partial x}, 0)$, with analogous expressions for the magnetic quantities. With the exception of the mean magnetic field, $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, all quantities are functions of $\mathbf{r} = (x, y, z)$ and time, although the z dependence is "slow" (Montgomery 1982; Zank & Matthaeus 1992). Note that the final term in each equation produces the long wavelength coupling of the 2D planes.

The above standard RMHD equations require augmentation if they are to be usefully applied within the context of the present model. We therefore introduce terms representing (in a volume-averaged sense) the forcing, reflection, and transmission of the waves. Energy is injected into a specified upward mode at rate F (corresponding to injection of kinetic plus magnetic energy at rate $F/4$), the outward propagating wave energy flux is reflected into inward propagating modes at rate R^-, with R^+ being the analogous rate for reflection of inward waves into outward. All outward modes experience a reduction in their energy at the rate T which simulates transmission of these waves out of the system. Denoting the energy (per unit mass) in outward waves by $E^- = \langle \mathbf{v}^2 - \mathbf{b}^2\rangle$ and that in inward modes by E^+, we add additional terms to Eqs. (1)–(2) so that the following energy equations are satisfied,

$$\frac{\mathrm{d}E^-}{\mathrm{d}t} \;=\; F - \left[R^- + T\right]E^- + R^+E^+, \quad (3)$$

$$\frac{\mathrm{d}E^-}{\mathrm{d}t} \;=\; R^-E^- - R^+E^+. \quad (4)$$

The lefthand sides of these equations represent the dynamics embodied in Eqs. (1)–(2). Note that while the reflection and transmission terms in Eqs. (3)–(4) have a simple form, the corresponding forms for inclusion in Eqs. (1)–(2) are more complicated.

It can be shown that the reflection rates $R^\pm$ are of order V_A/Δ, where Δ is a typical length for radial changes in the Alfvén velocity V_A (Hollweg 1981; Hollweg 1996; Velli 1993; Zhou & Matthaeus 1990a), while the transmission rate is $T \sim V_A/L$, with L the box length in the parallel direction (see Matthaeus et al. (1999) for further details). As noted above, RMHD dynamics requires $\tau \lesssim \tau_A$, or equivalently $Z/\lambda_\perp \gtrsim V_A/L$, where $\lambda_\perp$ is a characteristic length-scale for the quasi-2D component. Thus, for units where $\tau = 1$, as is convenient in simulations and phenomenologies, one should require that $T \leq 1$. Strong reflection then corresponds to $R > 1$, with weak reflection being the reverse.

4. RESULTS

We have performed a range of pseudospectral RMHD simulations, incorporating the extra terms representing wave driving, reflection, and transmission, as described above.

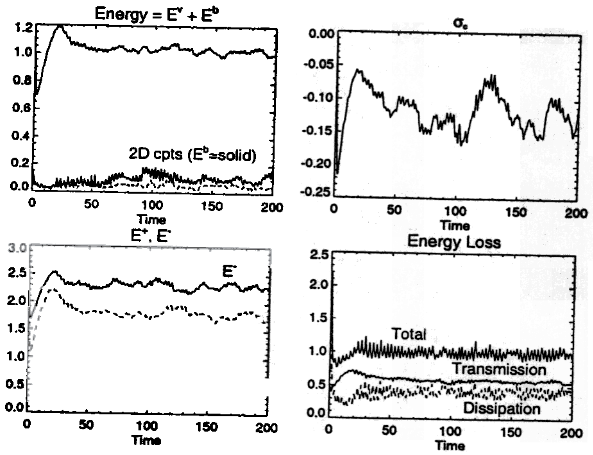

Figure 2. Time histories for energies and related quantities from a simulation with $F = 1$, $R = 0.5$, $T = 0.3$, and $\nu = \eta = 1/200$.

Figure 2 displays time histories of some useful quantities for a typical run. The resolution is 64 Fourier modes in the two transverse directions, and 4 Fourier modes in the vertical ($\mathbf{B}_0$) direction. The physical parameters for this particular run are $\eta = \nu = 1/200$, $F = 1$, $R = R^+ = R^- = 0.5$, and $T = 0.3$; time is measured in (quasi-2D) eddy-turnover times. The initial conditions are chosen approximately flat in k_z, and only transverse wavenumbers lying between 2 and 6 are excited. The single driven mode has $\mathbf{k} = (1, 1, 1)$.

From the figure it is clear that the dynamics has (at least) three phases. First, there is a rapid adjustment of the flow while turbulent correlations are established. This takes place over the first 1–2 eddy-turnover times. Since coronal hole plasma is assumed to be in an (approximately) steady state as far as heating and acceleration are concerned, this phase is probably not particularly relevant to the sun's atmosphere at present. In the second phase, the driving and reflection terms appear to be the controlling effects, with a steady ramping up of the total energy taking place for a few tens of eddy-turnover times. Note that the average energy levels for the (strict) 2D component hardly change at all during this phase. Shortly thereafter, the flow enters a state characterised by statistically steady values for many of the bulk quantities, such as the total energy, dissipation and transmission rates, and normalised cross helicity [$\sigma_c = (E^+ - E^-)/(E^+ + E^-)$]. Sizable fluctuations superimposed on the average steady levels are evident. We anticipate that the amplitude, duration, and frequency of occurrence of these events will depend in a non-trivial way on the Reynolds numbers and forcing, reflection, and transmission rates. Such events are also observed in simulations of the heating of closed field line regions (Einaudi et al. 1996; Dmitruk & Gómez 1997), suggesting perhaps that the heating of magnetically open and magnetically confined regions is more akin than has been previously thought. This similarity is discussed further in the final section.

The plots of $E^{\pm}$ and σ_c reveal that the majority of the energy is, persistently, associated with outward mode fluctuations, although the preponderance is not large. It is instructive to compare the steady levels achieved in this run with the corresponding values obtained when solving the steady—and thus purely linear—form of Eqs. (3)–(4). For this case we obtain the steady values $E^{\pm} = F/T$ and $\sigma_c = 0$. Thus, the nonlinear interactions break the up/down symmetry of the steady-state to energetically favour modes of the forced type.

Note that the first panel indicates that most of the energy is in "wave" modes ($k_z \neq 0$), showing that even when the 2D component is not energetically dominant, it can still provide an effective channel for the dissipation and heating. There is thus the intriguing possibility of the system undergoing phase transitions between states with heating and those without—depending on the values of the kinetic and magnetic Reynolds numbers, reflection and transmission strengths, and forcing strength and the initial 2D energy level. One might suggest—or more accurately at this stage, speculate—that such behaviour could lead to differences in the atmospheric dynamics of stars of various types and ages (see also Velli (1993)).

A fundamental question for models such as the present one concerns the efficiency of the heating process. That is, given the injection of a (statistically) steady wave flux at the coronal base, one would like to know how much of the associated energy is transmitted out of the system and how much ends up as heat. We have investigated this point for various values of the reflection rate R, holding $F = 1$ and $T = 0.3$ (*i.e.*, the same values as pertain to figure 2). The results are shown in figure 3. It is apparent that the heating rate approaches the energy loss due to transmission (from below) as R/T becomes large. At the other extreme, one observes that even for reflection rates of order 0.01, over 10% of the injected energy can show up as heat. Thus, weak reflection strengths—implying weak gradients in the mean density and magnetic field—are still able to catalyse effective heating of the plasma.

We note that approximately the same quantitative

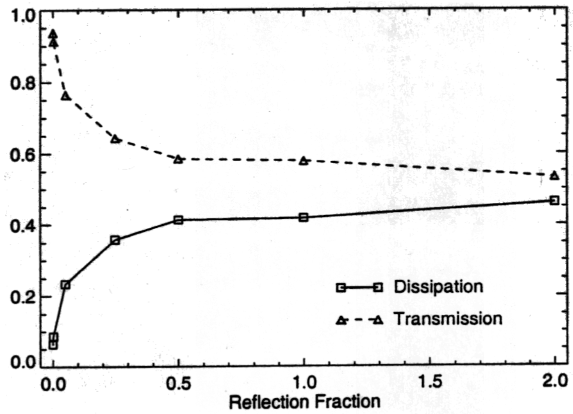

Figure 3. Steady-state levels for the transmitted wave flux and the total turbulent (viscous and resistive) dissipation rate as a function of reflection strength. Individual data points represent late-time ($t > 50$) averages for distinct simulations which have their forcing and transmission parameters held constant at $F = 1$ and $T = 0.3$. Note that the two curves are normalised so that they sum to unity.

behaviour is also observed for a recently developed phenomenology of the present model (Matthaeus et al. 1999). This consistency between the phenomenology and the simulations is encouraging and will bolster confidence in analytic results deriving from the former.

5. CONCLUSIONS

We have described a mechanism for heating open field line regions of the corona. The novel aspect of the model is that energy is extracted from Alfvén waves without requiring *direct* (*e.g.*, cyclotron) damping of the waves. Instead, counter-propagating waves couple with quasi-2D MHD turbulence to drive the latter. Energy in the quasi-2D modes cascades to small (perpendicular) scales where it is converted into heat. The initial population of upward waves develops a downward component through the action of (non-WKB) reflection of the upward waves off gradients in the mean fields.

The feasibility of the mechanism is investigated here within the context of reduced MHD simulations, with terms representing the spatially inhomogeneous wave driving, reflection, and transmission also included. The results are promising, providing "proof of principle" support for the model, with heating efficiencies of 50% (relative to the input wave power) being relatively easy to achieve. Indeed, even for unfavourable values of the reflection and transmission rates, the heating efficiency can still exceed 10%.

It is encouraging to note that the simulation results are in excellent qualitative and good quantitative agreement with those produced by a nonlinear phenomenology developed for the same model. In fact, the phenomenology is slightly more general than the simulations in one regard, in that the driving waves are not restricted to be of *reduced* MHD type (Matthaeus et al. 1999). On the other hand the RMHD simulations represent a much more stringent test of the viability of the model in the following way: the phenomenological treatment essentially assumes that the turbulence always dissipates at the rate associated with fully developed turbulence. This represents somewhat of a bias in favour of turbulent heating since all fluctuations contribute to the turbulent decay in the same way. However in the RMHD model the turbulent transfer and heating must persist on its own, despite potential solutions which involve attenuation of the nonlinear interactions to leave a simple wave propagation model. As demonstrated here, the solutions prevailing manifestly maintain the turbulent couplings and associated heating.

Having shown that the mechanism works the question then arises as to its quantitative relevance in the corona. The key quantities to estimate are $\tau_{\rm NL}$, T, R, and F. Velli (1993) has investigated the linear behaviour of Alfvén waves propagating through isothermal atmospheres, both with and without a wind. Here however, we proceed by calculating rough estimates for the above parameters using observational values and the model equations. For the corona, $\lambda_\perp$ is probably no bigger than the super-granulation scale (30 Mm), and perhaps as small as one tenth of this (Axford & McKenzie 1997), while various observational estimates for the non-thermal components of coronal emission linewidths suggest that $Z \approx 30$–$50\,{\rm km/s}$ (*e.g.,* Chae et al. 1998). Thus, $\tau_{\rm NL} \sim 100$–$1000\,{\rm s}$, and driving waves which are RMHD in character should have periods greater than this. Nondimensionalising in terms of $\tau_{\rm NL}$ and assuming driving wave periods of 100–200 s, Alfvén speeds of 1–$2 \times 10^3\,{\rm km/s}$, and a box length $L \sim 0.5\,{\rm R_s}$, one obtains values for the transmission rate of $T \sim 0.3$–1.2. Since $R \sim V_A/\Delta$ is structurally identical to T, in the same units one will have $R > T$ if the Alfvén speed changes on radial scales smaller than L, and conversely for $R < T$. As the scale height for the Alfvén speed is expected to be of order ${\rm R_s}$, $R \sim T$ may be typical. For such cases (see figure 3) about 40% heating efficiency is expected on the basis of the present model.

We have already noted above that there are parallels between the heating of open and closed field line regions. Many closed field line heating models also rely on the interaction of counter-moving disturbances (generated, for example, by photospheric footpoint motions at each end of a coronal loop). As in the present model, these waves could then couple to quasi-perpendicular fluctuations with a subsequent cascade of the energy to small-scales and eventual conversion into heat (*e.g.,* Velli 1996; Einaudi et al. 1996; Dmitruk & Gómez 1997; Priest et al. 1998). There is then the suggestion that the same heating mechanism is active in both types of field line regions, with the major difference being the way in which the counter-propagating Alfvénic fluctuations are generated.

Finally, there is the question of the exact nature of the dissipation. Our mechanism has been presented within the context of fluid models. Naturally kinetic processes will govern the actual damping of the small-scale quasi-2D turbulence generated by the highly anisotropic spectral transfer, and one would like to understand these processes in detail. Some possible mechanisms have been investigated by Leamon et al. (1998a,b) for solar wind fluctuations. These studies are also applicable to coronal fluctuations and include examination of the roles of ion-cyclotron damping, Landau damping, and nonlinear kinetic processes such as beam instabilities and/or mode conversion and damping. Further examination of these possibilities is in progress.

ACKNOWLEDGMENTS

This work has been supported by the UK PPARC (GR/K98711), and NASA through the SR+T (NAG5-7164), HMGI (NAG5-6570), and SECT Programs (NAG5-8134).

REFERENCES

Axford W. I., McKenzie J. F., 1997, in Cosmic Winds and the Heliosphere, Arizona U. Press, 31

Chae J., Schühle U., Lemaire P., 1998, Astrophys. J. **505**, 957

Culhane J. L., 1998, Adv. Space Res. **19**, 1839

Dmitruk P., Gómez D. O., 1997, Astrophys. J. **484**, L83

Einaudi G., Velli M., Politano H., Pouquet A., 1996, Astrophys. J. **457**, L113

Grall R. R., Coles W. A., Klinglesmith M. T., Breen A. R., Williams P. J. S., Markkanen J., Esser R., 1996, Nature **379**, 429

Guhathakurta M., Sittler E. C., 1999, J. Geophys. Res., in press

Habbal S. R., Esser R., Guhathakurta M., Fisher R. R., 1995, Geophys. Rev. Lett. **22**, 1465

Hollweg J. V., 1981, Solar Phys. **70**, 25

Hollweg J. V., 1996, in D. Winterhalter, J. T. Gosling, S. R. Habbal, W. S. Kurth, M. Neugebauer (eds.), Solar Wind Eight, AIP, New York, 327

Hossain M., Gray P. C., Pontius Jr. D. H., Matthaeus W. H., Oughton S., 1995, Phys. Fluids **7**, 2886

Hossain M., Vahala G., Montgomery D., 1985, Phys. Fluids **28**, 3074

Kinney R., McWilliams J. C., 1998, Phys. Rev. E **57**, 7111

Leamon R. J., Smith C. W., Ness N. F., Matthaeus W. H., Wong H. K., 1998a, J. Geophys. Res. **103**, 4775

Leamon R. J., Matthaeus W. H., Smith C. W., Wong H. K., 1998b, Astrophys. J. **507**, L181

Matthaeus W. H., Ghosh S., Oughton S., Roberts D. A., 1996, J. Geophys. Res. **101**, 7619

Matthaeus W. H., Lamkin S. L., 1986, Phys. Fluids **29**, 2513

Matthaeus W. H., Oughton S., Ghosh S., Hossain M., 1998, Phys. Rev. Lett. **81**, 2056

Matthaeus W. H., Zank G. P., Oughton S., Mullan D. J., 1999, Astrophys. J. **in press**

McKenzie J., Banaszkiewicz M., Axford W. I., 1995, Astron. Astrophys. **303**, L45

Montgomery D. C., 1982, Physica Scripta **T2/1**, 83

Montgomery D. C., Turner L., 1981, Phys. Fluids **24**, 825

Moore R. L., Musielak Z. E., Suess S. T., An C.-H., 1991, Astrophys. J. **378**, 347

Musielak Z. E., Fontenia J. M., Moore R. L., 1992, Phys. Fluids B **4**, 13

Oughton S., Ghosh S., Matthaeus W. H., 1998, Phys. Plasmas **5**, 4235

Oughton S., Priest E. R., Matthaeus W. H., 1994, J. Fluid Mech. **280**, 95

Priest E. R., Foley C. R., Heyvaerts J., Arber T. D., Culhane J. L., Acton L. W., 1998, Nature **393**, 545

Shebalin J. V., Matthaeus W. H., Montgomery D., 1983, J. Plasma Phys. **29**, 525

Sittler E. C., Guhathakurta M., 1999, Astrophys. J., in press

Tu C.-Y., Marsch E., 1997, Solar Phys. **171**, 363

Ulmschneider P., Priest E. R., Rosner R., 1991, Mechanisms of Chromospheric and Coronal Heating, Springer, Berlin

Velli M., 1993, Astron. Astrophys. **270**, 304

Velli M., 1996, in D. Winterhalter, J. T. Gosling, S. R. Habbal, W. S. Kurth, M. Neugebauer (eds.), Solar Wind Eight, AIP, New York, 28

Zank G. P., Matthaeus W. H., 1992, J. Plasma Phys. **48**, 85

Zhou Y., Matthaeus W. H., 1990a, J. Geophys. Res. **95**, 10 291

Zhou Y., Matthaeus W. H., 1990b, J. Geophys. Res. **95**. 14 863