University of Waikato
Department of Mathematics NumberTheory Assignment 4

Due Monday 19th May 2008.

1. Verify that $(350|19) = -1$.

2. Does $x^2 \equiv 13 \bmod 23)$ have a solution ? If so find it.

3. Show that the number 3 is a quadratic non-residue for all primes of the form $p = 4^n + 1$.

4. Show that if $p \equiv 7 \bmod 8$ then

$$p \mid 2^{\frac{p-1}{2}} - 1.$$

Hence find a factor of $2^{83} - 1$.

5. Evaluate $(3|p)$ for the primes p in the residue classes 1,5,7 or 11 mod 12 (it has to be one of these). Can you express your results as a neat formula?

6. Find the finite continued fraction expansion for 8/17. If $\alpha = [1, 2, 3, 4, 5]$ evaluate α as a rational number and find the distance between α and the approximation $\beta = [1, 2, 3, 4]$.

7. Verify the first three terms in the infinite continued fraction expansion for $\pi = [3, 7, 15, 1, 292, ...]$ using the integer part function IntegerPart, and using Mathematica's $N[Pi, 40]$, find an upper bound for the distance between π and the rational approximations given by using 2,3,and 4 convergents.

8. Verify that the fundamental solutions for the Pell equation $x^2 - 22y^2 = 1$ is $x = 197, y = 42$. You may assume the continued fraction expansion for $\sqrt{22} = [4, \overline{1, 2, 4, 2, 1, 8}]$.

Kevin Broughan
12th May 2008