University of Waikato
Department of Mathematics
math310-08A: Number Theory Assignment 3
Due Monday 7th April 2008. Normal hand-in instructions.

1. Determine which of the following integers can be written as the sum of two squares:

$$150, 151, 152, 153, 154.$$

2. If $(a, b) = 1$ and $ab = c^n$ prove that both a and b are n'th powers.

3. How many pythagorean triangles are there with hypotenuse less than 50 ? List them.

4. Prove that if m and n can be expressed as the sum of two squares and $m \mid n$ then n/m can be expressed as the sum of two squares.

5. From $53 = 7^2 + 2^2 + 0^2 + 0^2$ find a representation of 18179 as a sum of 4 squares. Hint $18179 = 7^3 \cdot 53$.

6. Show that if $4 \mid x^2 + y^2 + z^2$ then x, y, z are all even.

7. Show that if $n \equiv 7 \bmod 8$ then n cannot be written as the sum of three squares.

8. Use the result of 6. and 7. to show that if $n = 4^e(8k + 7)$ for some $e \geq 0$ and $k \geq 0$ then n is not able to be written as the sum of three squares. Show that the smallest n which cannot be written as the sum of three squares is of this form.

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