# University of Waikato
# Department of Mathematics
# Number Theory Assignment 2

## Due Wed 23rd March 2005

Hand in your work, clearly labelled with your name, id number and the number of the assignment, through the slot "Number Theory" outside the Mathematics office.

**math54.314B: 1,2,3,5,6,7,8.**
**math54.4/514B: 1-10.**

1. Evaluate $\sigma(2^3 3^2 5)$ by hand (usng the product formula).

2. Write a short paragraph (say 5-10 lines) about Mersenne or Fermat numbers giving some history and more recent results.

3. You want the know for some applications the highest power of a given prime p which divides into a binomial coefficient $\binom{2n}{n}$ so can use the result of Lagrange we proved (since the binomial coefficient is made up using factorials). Try this explicitly by hand for $p = 3$ and $n = 5$ and then using Lagrange for $p = 3$ and $n = 20$.

4. Prove that for each prime $p$ in the range $n < p < 2n$, $\binom{2n}{n} \equiv 0 \bmod p$ and$\binom{2n}{n} \not\equiv 0 \bmod p^2$.

5. Make up a multiplication and an addition table for the residue class ring $\mathbb{Z}_{12}$ and give the inverse class for each element which has an inverse (there are $\varphi(12)$ of these).

6. Verify the equation $n = \sum_{d|n} \phi(d)$ in the special case $n = 18$.

7. Show that for each natural number $n > 2$, one of the numbers $n, n+1$ must be composite. Show that, for $n > 3$, one of the numbers $n, n+2, n+4$ must be composite. Find the smallest $n$ such that $n+1, n+2$, and $n+3$ are all composite.

8. Show that the divisor function $d(n)$ (the number of divisors of n) is multiplicative by expressing it as the Dirichlet product of two very simple multiplicative functions.

9. If for $n \geq 0$, $F_n$ is the n'th Fermat number, prove that

$$F_n - 2 = \prod_{j=0}^{n-1} F_j.$$

Hint: use induction.

10. Prove that for all $n \in \mathbb{N}$, $\sigma(n)\phi(n) \leq n^2$.

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**16th March 2005**