University of Waikato
Department of Mathematics
Number Theory Assignment 1 (revised)

Due Wed 16th March 2005

Hand in your work, clearly labelled with your name, id number and the number of the assignment, through the slot "Number Theory" outside the Mathematics office.

math54.314B: 1,3,4,6,7,9,10.
math54.4/514B: 1-10.

1. Prove that if a and b are odd integers then $a^2 - b^2$ is divisible by 8.

2. Find all the multiples of 8 ($n = 8m$) which are expressible in the form $n = a^2 - b^2$.

3. Use a sieve to find all of the primes in the range [2, 144].

4. Factor the number 7! (seven factorial) completely and express in standard form.

5. Prove that for each natural number n if p, q are two distinct primes dividing $n!$ with $p < q$ then the power of p which appears in the standard factorization of $n!$ is greater than or equal to the power of q so appearing.

6. Use the Euclidean algorithm to find the gcd (52,39) and hence find x and y so that $52x + 39y = (52, 39)$.

7. Find the general solution for the integer equation $11x + 5y = n$ and for $n = 40$ a positive solution. List as many as possible values of n, $0 \leq n \leq 39$, for which there is a positive solution.

8. Prove that for all integers a and b that $(a + b, a - b) \geq (a, b)$

9. It is a long standing conjecture that no odd number is perfect. Prove that a power of an odd prime is never perfect and that the product of two **odd** distinct primes is never perfect.

10. Prove that if (x_o, y_o) is a solution to the linear equation in integers $ax - by = 1$ then the area of the triangle with vertices $(0, 0), (b, a), (x_o, y_o)$ is 1/2 and that the number of points with integer coordinates on the line joining $(0, 0)$ to (b, a) is the GCD (a, b).

Kevin Broughan
15th March 2005