# University of Waikato
# Department of Mathematics
# Number Theory Assignment 1 Corrected

## Due Wed 10th March 2008

Hand in your work, clearly labelled with your name, id number and the number of the assignment, through the slot "Algebra and Number Theory" outside the Mathematics office.

1. Use a sieve to find all of the primes in the range [2, 144].

2. Use the Euclidean algorithm to find the gcd (52,39) and hence find x and y so that $52x + 39y = (52, 39)$.

3. Find the general solution for the integer equation $11x + 5y = n$ and for $n = 40$ a positive solution. List as many as possible values of $n$, $0 \leq n \leq 39$, for which there is a positive solution.

4. Prove that for all integers $a$ and $b$ that $(a + b, a - b) \geq (a, b)$

5. It is a long standing conjecture that no odd number is perfect. Prove that a power of an odd prime is never perfect and that the product of two **odd** distinct primes is never perfect.

6. Prove that if $a$ and $b$ are odd integers then $a^2 - b^2$ is divisible by 8.

7. Find all the multiples $n$ of 8 ($n = 8m$) which are expressible in the form $n = a^2 - b^2$.

8. Let $m = n(n + 1)(n + 2)$ be the product of any three successive integers. Prove that $m$ is divisible by 6.

Kevin Broughan
**5th March 2008**