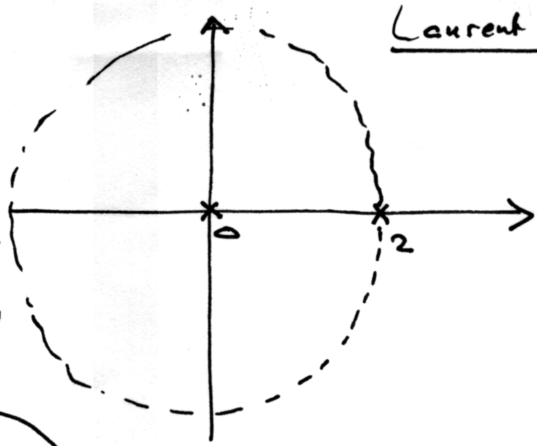
Laurent Series Example

311A

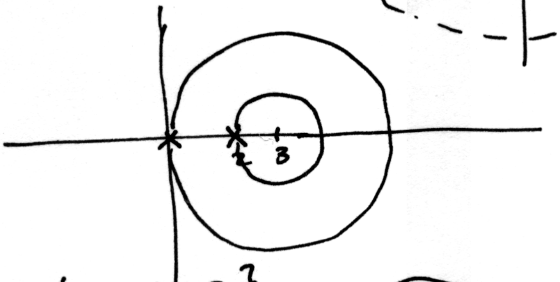
$$f(z) = \frac{1}{z(1-z)}$$



centre $= 0$

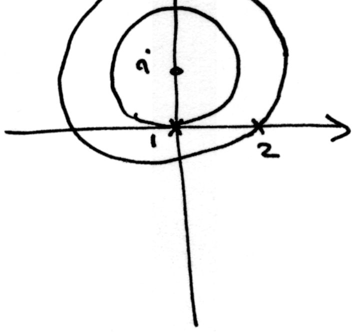
$A_1 = \{z \mid 0 < |z| < 2\}$ $r=0, R=2$

$A_2 = \{z : |z| > 2\}$ $r=2, R=\infty$



$A_3 = \{z : 1 < |z-3| < 3\}$

centre $= 3$



$A_4 = \{z : 1 < |z-i| < \sqrt{5}$

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① $f(z) = \dfrac{1}{2z(1-\frac{z}{2})}$ $\left|\frac{z}{2}\right| < 1 \iff |z| < 2$ ✓ so $0 < |z| < 2$.

$$= \frac{1}{2}\cdot\frac{1}{z}\left[\sum_0^\infty \frac{z^n}{2^n}\right] = \frac{1}{2}\cdot\frac{1}{z}\left[1 + \frac{z}{2} + \frac{z^2}{4} + \cdots + \frac{z^j}{2^j}\right]\cdots$$

$$= \frac{1/2}{z} + \frac{1}{2} + \frac{z}{4} + \cdots + \frac{z^{j-1}}{2^{j+1}} +$$

$c_{-1} = 1/2$ $c_0 = \frac{1}{2}$ $c_1 = \frac{1}{4}$ $c_n = \frac{1}{2^{n+2}}$ etc.

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② $|z| > 2$ $f(z) = \dfrac{1}{z(2-z)} = \dfrac{1}{-z^2(1-\frac{2}{z})}$

still centre $0 \Rightarrow \frac{1}{z^m}$ or z^n

$\iff \left|\frac{2}{z}\right| < 1$

$$= -\frac{1}{z^2}\left[1 + \frac{2}{z} + \frac{4}{z^2} + \cdots\right]$$

$$= -\frac{1}{z^2} - \frac{2}{z^3} - \frac{4}{z^4} - \cdots$$

$c_{-2} = -1$ $c_{-3} = -2$ $c_{-4} = -4$

rest are 0.