The University of Waikato
Department of Mathematics

Advanced Calculus math311-04A 2004 Complex Assignment 4

Due Friday 4th June: Please hand back your completed assignment through the slot outside the Mathematics Office G3.19.

It should be written up neatly and on no more than four sides of an A4 page or the equivalent.

1. Find poles and residues at the poles of

$$f(z) = \frac{z+4}{(z+i)^2(z-2)}.$$

2. Evaluate the complex integral

$$\int_\Gamma f(z)dz$$

where $f(z)$ is the function defined in question 1 above, and where Γ is a piecewize linear path joining in order the following points: $\{-1-2i, 1-2i, 1-i, 3-i, 3+i, -1+i, -1-2i\}$ **.**

3. Prove that, if $m > 0$

$$\int_0^\infty \frac{\cos mx}{(x^2+1)^2}dx = \frac{\pi e^{-m}(1+m)}{4}.$$

4. Use the Residue Theorem with a suitable contour in the complex plane to show that:

$$\int_0^\infty \frac{\sin ax}{e^x+1} = \frac{1}{2a} - \frac{\pi}{2\sinh \pi a}.$$

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