The University of Waikato
Department of Mathematics

Advanced Calculus math311-03A 2003 Complex Assignment 3

Due Tuesday 6'th May: Please hand back your completed assignment through the slot outside the Mathematics Office G3.19. It should be written up neatly and on no more than four sides of an A4 page or the equivalent.

1. If Γ is the unit circle show that

$$\int_0^{2\pi} \frac{\cos^2\theta}{5-4\sin\theta} d\theta$$

can be expressed as the integral of a rational function of z. Comment on how you might evaluate this integral (but don't do it). Hint: on Γ, $z = e^{i\theta}$ so $\cos\theta = (z+1/z)/2$ etc.

2. Use the Fundamental theorem to write down the value of the integral

$$\int_0^{a+ib} e^z dz.$$

Then equate the answer with the one obtained by parametric evaluation along the straight line from 0 to $a+ib$, $\Gamma(t) = t(a+ib)$, to derive the real integral

$$\int_0^1 e^{ax}\cos(bx)dx = \frac{a(e^a\cos b - 1) + be^a \sin b}{a^2+b^2}.$$

Comment on whether the same method might work for the important integral: $\int e^{ax^2}\cos(bx)dx$.

3. Find the center, radius of convergence and circle of convergence

of the power series

$$f(z) = \sum_{n=1}^{\infty} \frac{n(-1)^n(z-i)^n}{4^n}.$$

Examine the convergence, or otherwise, of the series on the circle of convergence.

Kevin Broughan
29th April 2003