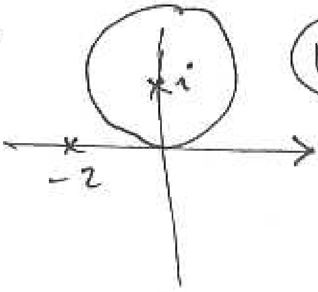
matt 311 - 07 A   Advanced Calculus Ass 3   Solutions   Pg 1

(1)



$$\frac{z+4}{(z-i)(z+2)} = \frac{\cancel{z+2+2}\; z-i+4+i}{(z-i)(z+2)}$$

$$= \frac{1}{z+2} + \frac{(4+i)}{(z-i)(z+2)}$$

$$\frac{1}{(z-i)(z+2)} = \frac{1/(2+i)}{z-i} - \frac{1/(2+i)}{z+2}.$$

$$\Rightarrow \oint_\Gamma \frac{z+4}{(z-i)(z+2)}\,dz = \oint_\Gamma \frac{dz}{z+2} - \frac{1}{(2+i)}\oint_\Gamma \frac{dz}{z+2}.$$

$$+ \frac{(4+i)}{(2+i)} \int_\Gamma \frac{dz}{z-i}. \qquad \xrightarrow{z=i+e^{i\theta}} \int_0^{2\pi} \frac{ie^{i\theta}d\theta}{e^{i\theta}} = 2\pi i$$

$$= \frac{4+i}{2+i}\,2\pi i$$

$$= \frac{(4+i)(2-i)}{(2+i)(2-i)}\,2\pi i$$

$$= \frac{8+1-2i}{4+1}\,2\pi i$$

$$= \frac{2\pi}{5}(2+9i)$$

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(1)' cent $= -i$

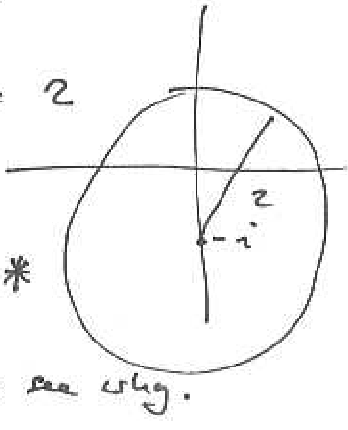
$$R_f = \lim_{n\to\infty} \frac{|a_n|}{|a_{n+1}|} = \lim_{n\to\infty} \frac{\left|\frac{(-1)^n}{n\,2^n}\right|}{\left|\frac{(-1)^{n+1}}{(n+1)\,2^{n+1}}\right|}$$

$$= \lim_{n\to\infty} \frac{(n+1)\,2^{n+1}}{n\,2^n}$$

$$= \lim_{n\to\infty} \left(1+\frac{1}{n}\right)2 = 2$$

* $f(z) = \sum b_n$ does not converge absolutely at any point on the circle of convergence.

So circle of conv is $\boxed{z = -i + 2e^{i\theta}}$

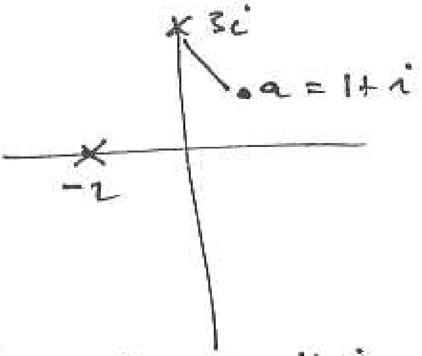


on the circle $f(z) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} e^{ni\theta}$. $|b_n| = \frac{1}{n} \Rightarrow$ *

This converges conditionally, but it is not easy to see why.

RS2

(3) $f(z) = \dfrac{z}{(z-3i)(z+2)}$ has simple pole at $\{3i, -2\}$.



If a is the centre, $a = 1+i$, $|a-3i| = |1+i-3i| = |1-2i|$

$= \sqrt{1^2+(-2)^2} = \sqrt{5}$

and $|a-(-2)| = |1+i+2| = |3+i| = \sqrt{9+1} = \sqrt{10}$

& $\sqrt{5} < \sqrt{10}$ $\therefore$ $R_f = \sqrt{5}$.

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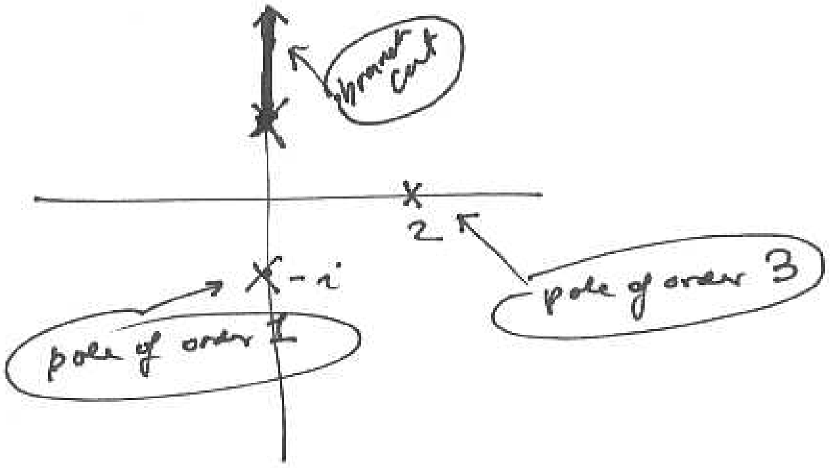
(4) $f(z) = 0$ when the numerator is 0 i.e. $z = 0$ (a double)

and $z = i$ (a branch pt singularity).

It has poles where the denominator is 0 i.e. $z = -i$, a simple pole and $z = 2$, a pole of order 3.



To make f single valued, and holomorphic, we should remove the branch point and a line from $z = i$ to ∞, from its domain e.g. $L = \{z : \mathrm{Im}(z) \geq 1\}$. Then the domain of f is

$$\mathbb{C} \setminus (L \cup \{-i, 2\})$$