

The University of Waikato  
Department of Mathematics

Advanced Calculus math311-03A 2003 Complex Assignment 2

Due Tuesday 8<sup>th</sup> April: Please hand back your completed assignment through the slot outside the Mathematics Office G3.19. It should be written up neatly and on no more than four sides of an A4 page or the equivalent.

1. Let  $f(z) = 3z^2 - 8z + 1$ . Use the Cauchy-Riemann equations to show that  $f$  is holomorphic (analytic) on  $\mathbb{C}$  and find the derivative as a function of  $z$  using the expression  $f'(z) = u_x + iv_x$ .

2. Use  $\epsilon - \delta$  to show that

$$\lim_{z \rightarrow i} \frac{2}{z - i}$$

does not exist. Then analyse the contours of the real and imaginary parts of the expression in the neighbourhood of  $z = i$ .

3. Derive the identity  $\log(z_1 z_2) = \log(z_1) + \log(z_2)$ .

4. Use Cauchy's theorem and Cauchy's integral formula to evaluate

$$\oint_{\Gamma} \frac{dz}{z^2 - 2z}$$

where  $\Gamma$  is the circle  $\{z : |z - 2| = 1\}$ . Then try the same problem using the parametric definition of the complex integral.

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