

The University of Waikato  
Department of Mathematics

Advanced Calculus math311-04A 2004 Complex Assignment 1

Due Thursday 13th May: Please hand back your completed assignment through the slot outside the Mathematics Office G3.19.

It should be written up neatly and on no more than two sides of an A4 page or the equivalent.

1. Sketch the set of points  $A$  consisting of all  $z$  in the complex plane for which  $2 \leq |z - i| < 4$  and mark the interior, closure and boundary. Is  $A$  open, connected? Is the boundary of  $A$  connected? Is  $A$  a region?

2. Let  $f(z) = z^2 - 3z + 1$ . Use the Cauchy-Riemann equations to show that  $f$  is holomorphic (analytic) on  $\mathbb{C}$  and find the derivative as a function of  $z$  using the expression  $f'(z) = u_x + iv_x$ .

3. Use the multidirectional nature of the complex limit to show that

$$\lim_{z \rightarrow -i} \frac{1}{z + i}$$

does not exist. Then analyse the contours of the real part of the expression in the neighbourhood of  $z = -i$ .

4. Derive the identity  $e^{-z} = \frac{1}{e^z}$ , using the real and imaginary parts and complex inversion.

Kevin Broughan  
6th May 2004