

The University of Waikato
Department of Mathematics

Advanced Calculus 0654.311A 2003 Complex Assignment 1

Due Monday 24th March: Please hand back your completed assignment through the slot outside the Mathematics Office G3.19.

It should be written up neatly and on no more than two sides of an A4 page or the equivalent.

1. Show that if $a_n = ni^n$ then $\lim_{n \rightarrow \infty} a_n$ does not exist.

2. Use limit theorems to show that

$$\lim_{n \rightarrow \infty} \frac{(2n + 4i - 3)(n - 1)}{in^2 - in + 1 - 3i} = -2i.$$

3. Sketch the set of points A consisting of all z in the complex plane for which $1 < |z - 2i| < 2$ and find the interior, closure and boundary. Is A open, connected? Is the boundary of A connected? Is A a region?

4. Find an unbounded and connected set in the complex plane on which the function $f(z) = e^z$ is bounded.

5. Use an $\epsilon - \delta_\epsilon$ proof to show that

$$\lim_{z \rightarrow 1+i} z^2 + 1 = 2i + 1.$$

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17 March 2003