Solutions   Math311-07A   Complex Assignment 1

1) $z_1 = r_1 e^{i\theta_1}$ $\Rightarrow$ $z_1 z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$
$z_2 = r_2 e^{i\theta_2}$

so $|z_1 z_2| = r_1 r_2 = |z_1|\cdot|z_2|$.

2) $z(\bar{z}+2) = 3$. Let $z = x+iy$ $\Rightarrow$

$(x+iy)(x+2-iy) = 3$

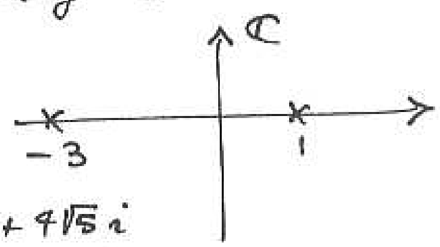
$\Leftrightarrow$ $x^2+y^2+2x + i(y(x+2) - xy) = 3$

$\Rightarrow$ $x^2+y^2+2x = 3$ and $2y = 0$ $\Rightarrow$

$x^2+2x-3 = 0$ and $y = 0$

$\Rightarrow$ $(x+3)(x-1) = 0$ and $y = 0$ so the points

are $(1,0)$ and $(-3,0)$



3) Let $\frac{\omega}{2} = x+iy$ satisfy $\omega^2 = 8+4\sqrt{5}i$
$= 4(2+\sqrt{5})i$

$\Rightarrow$ $\left(\frac{\omega}{2}\right)^2 = 2+\sqrt{5}i$

$\Rightarrow$ $x^2-y^2+2ixy = 2+\sqrt{5}i$ $\Rightarrow$ $x^2-y^2 = 2$ and $2xy = \sqrt{5}$ (*)

$\Rightarrow$ $x^2 - \left(\frac{\sqrt{5}}{2x}\right)^2 = 2$ $\Rightarrow$ $4x^4 - 5 = 8x^2$
or $4x^4 - 8x^2 - 5 = 0$
$(2x^2+1)(2x^2-5) = 0$.

But $x \in \mathbb{R}$ $\Rightarrow$ $x^2 = \frac{5}{2}$ $\Rightarrow$ $x = \pm\sqrt{\frac{5}{2}}$ &, by (*) $y = \frac{\pm\sqrt{5}}{2}\sqrt{\frac{2}{5}}$
$= \pm\frac{1}{\sqrt{2}}$

Hence $\omega = 2(x+iy)$
$= 2\left(\pm\sqrt{\frac{5}{2}} + \pm i\frac{1}{\sqrt{2}}\right) = \pm\sqrt{2}(\sqrt{5}+i)$ //

4) $f(z) = z^3+2z = (x+iy)^3 + 2x + 2iy$
$= x^3 + 3x^2(iy) - 3xy^2 - iy^3 + 2x + 2iy$
$= (x^3 - 3xy^2 + 2x) + i(3x^2y - y^3 + 2)$
$= u + iv$

so $\frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 2$ $\frac{\partial v}{\partial y} = 3x^2 - 3y^2 + 2 = \frac{\partial u}{\partial x}$

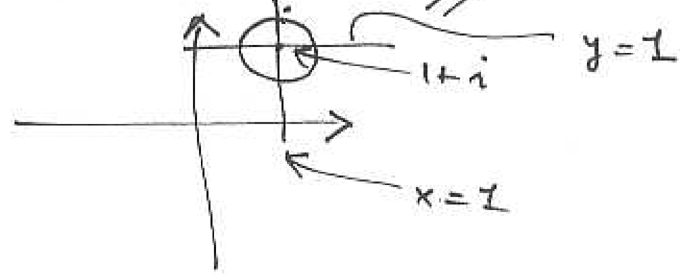
$\frac{\partial u}{\partial y} = -6xy$ $\frac{\partial v}{\partial x} = 6xy = -\frac{\partial u}{\partial x}$ on $\mathbb{C} = \mathbb{R}^2$

(over)

4 (cont) Hence the Cauchy-Riemann equations are satisfied on $\mathbb{C}$ so f is holomorphic on $\mathbb{C}$, or <u>entire</u>.

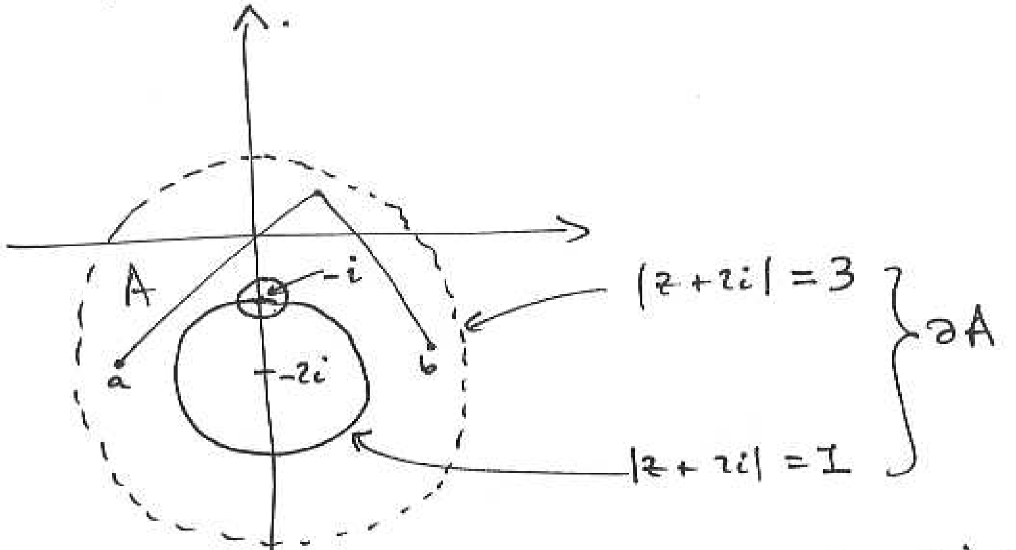
$$f'(z) = u_x + i v_x = 3x^2 - 3y^2 + 2 + 6ixy$$
$$= 3(x^2 - y^2 + 2ixy) + 2 = 3z^2 + 2. //$$

5 $$\lim_{z \to 1+i} \frac{1}{z-1-i}$$



$$= \lim_{\substack{x \to 1 \\ y = 1}} \frac{1}{x+iy-1-i}$$
$$= \lim_{\substack{x \to 1 \\ y = 1}} \frac{1}{x-1} \quad \nexists$$ hence the limit does <u>not</u> exist.

6



A is the set of points between two circles centre $-2i$, an <u>annulus</u>.

$\partial A = \{z \mid |z+2i| = 3\} \cup \{z \mid |z+2i| = 1\}$, ie. bdry pts of A & $\mathbb{C} \setminus A$,

$A^\circ = \{z \mid 1 < |z+2i| < 3\}$, the biggest open subset of A,

$\bar{A} = \{z \mid 1 \leq |z+2i| \leq 3\}$, each pt. is a limit of pts of A.

A is <u>not</u> open because there are many points, e.g. $-i$, for which no surrounding disk fits inside A.

A is connected, since any two point can be joined by a piecewise linear path eg. a & b above.

A is <u>not</u> a region — it is connected, but not open.

A° is a region.