

The University of Waikato
Department of Mathematics

Advanced Calculus math311-07A 2007 Complex Assignment 1

Due Friday 4th May: Please hand back your completed assignment through the slot outside the Mathematics Office G3.19.

It should be written up neatly and on no more than two sides of an A4 page or the equivalent.

1. Prove if z_1, z_2 are any complex numbers that $|z_1 z_2| = |z_1| |z_2|$.
2. Find and sketch the locus of points z in the complex plane such that $z(\bar{z} + 2) = 3$.
3. Find the 2 complex square roots of $8 + 4\sqrt{5}i$.
4. Let $f(z) = z^3 + 2z$. Use the Cauchy-Riemann equations to show that f is holomorphic (analytic) on \mathbb{C} and find the derivative as a function of z using the expression $f'(z) = u_x + iv_x$.

5. Use the multidirectional nature of the complex limit to show that

$$\lim_{z \rightarrow 1+i} \frac{1}{z - 1 - i}$$

does not exist.

6. Sketch the set of points A consisting of all z in the complex plane for which $1 \leq |z + 2i| < 3$ and mark the interior, closure and boundary. Is A open, connected? Is the boundary of A connected? Is A a region?

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27th April 2007