

**The University of Waikato Department of  
Mathematics**

**Metric Spaces Workshop/Tutorial number 10 for 2nd  
October: math501-09B**

These problems are quite hard and might need some background research to solve.

**Inverse Function Theorem:**

1. Use the Inverse Function Theorem (IFT) to find the largest open subset of  $\mathbb{R}^2$  where the differentiable inverse of  $f : (x, y) \rightarrow (4x, y^2 - 2y + 1)$  exists, and find an expression for it.

2. Formulate the Implicit Function Theorem first for maps  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  and then more generally. See if there is a way of using the IFT to prove this theorem. Example  $f(x, y) = y - x^2$ .

**Uniform Approximation:**

3. Show that every continuous function on  $[0,1]$  can be uniformly approximated by piecewise linear functions. (You may assume -via the Stone Weierstrass (SW) theorem to come later, that every continuous function can be uniformly approximated by polynomials.) Now try and do it explicitly for

$$f(x) = |x - 1| + 2|x - 2| + 1$$

on  $[0, 3]$ .

4. If  $c > 0$  use Taylor's theorem to show we can approximate the absolute value function uniformly by polynomials on  $[-c, c]$  (used in the proof of the SW theorem).

2nd October 2009