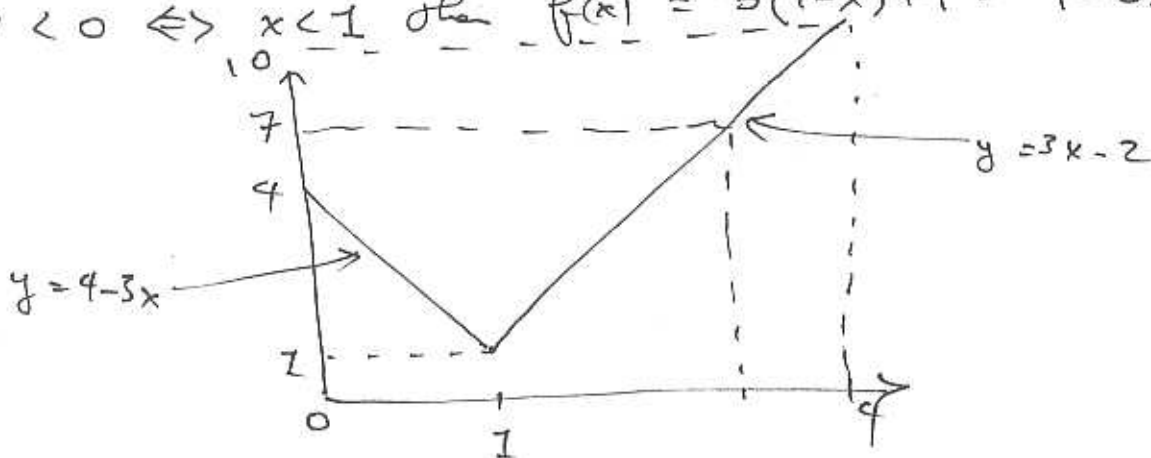


① $f(x) = 3|x-1| + 1$ on $[0, 4]$

If $x-1 \geq 0 \Leftrightarrow x \geq 1$ then $f(x) = 3(x-1) + 1 = 3x - 2$

If $x-1 < 0 \Leftrightarrow x < 1$ then $f(x) = 3(1-x) + 1 = 4 - 3x$



$f(0) = 4 - 3 \cdot 0 = 4$, $f(1) = 3|1-1| + 1 = 1$, $f(4) = 3 \cdot 4 - 2 = 10$

Hence $m = 1 \leq f(x) \leq 10 = M$ on $[0, 4]$ or
 $m = 1 \leq f(x) \leq 7 = M$ on $[0, 3]$

Solve $7 = f(x) = 3x - 2 \Rightarrow 3x = 9 \Rightarrow x = 3$.

② $f(0+h) = f(0) + f'(0)h + \frac{f''(0+\theta h)}{2!}h^2$

$f(x) = x^3 + 2x^2$

$f'(x) = 3x^2 + 4x$

$f''(x) = 6x + 4$

$h^3 + 2h^2 = f(h) = 0 + 0 \cdot h + \frac{1}{2}(6\theta h + 4)h^2$

$\Rightarrow \cancel{h^2}(h+2) = \frac{\cancel{h^2}}{2}(6\theta h + 4) = \cancel{h^2}(3\theta h + 2)$

$\Rightarrow h+2 = 3\theta h + 2 \Rightarrow \theta = \frac{1}{3}$

$f'(0) = 0$ + $f''(0) = 4 > 0 \Rightarrow (0, f(0))$ is a local min.

Inflection $f''(x) = 0 \Rightarrow 0 = 6x + 4 \Rightarrow x = -\frac{2}{3}$

so $(-\frac{2}{3}, f(-\frac{2}{3}))$ is a point of inflection.

3 Defn $U(f, P) = \sum_{i=1}^n M_i(f) \Delta x_i$ where

$M_i(f) = \sup \{ f(x) : x_{i-1} \leq x \leq x_i \}$, $\Delta x_i = x_i - x_{i-1}$,

$P = (x_0, x_1, \dots, x_n)$ $x_0 = a, x_n = b$.

By "f is Ri.Int. on [a,b]" we mean $\sup_P L(f, P) = \inf_P U(f, P)$

and then $\int_a^b f$ is the common value.

If $f(x) \geq 0 \Rightarrow \inf \{ f(x) : x_{i-1} \leq x \leq x_i \} \geq 0$

$\Rightarrow L(f, P) \geq 0 \forall P \Rightarrow \sup L(f, P) \geq 0 \Rightarrow \int_a^b f \geq 0$.

4 By "f is uniformly continuous on [a,b]" we mean $\forall \epsilon > 0 \exists \delta > 0$ such that $\forall x, y \in [a, b], |x - y| < \delta \Rightarrow |f(x) - f(y)| < \epsilon$.

If f is continuous but not uniformly continuous, $\exists \epsilon_0 > 0$ so.... see notes

Let $b > 0$ be fixed. By the M.V.Th. $f(x) - f(y) = f'(\xi)(x - y)$

so $|f(x) - f(y)| = |f'(\xi)| |x - y| \leq 4b |x - y|$ since $x < \xi < y \Rightarrow \xi \leq b$.

Hence if $\epsilon > 0$ is given we let $\delta = \frac{\epsilon}{4b}$, if $|x - y| < \delta \Rightarrow$

$|f(x) - f(y)| \leq 4b \delta = 4b \frac{\epsilon}{4b} = \epsilon \Rightarrow f$ is unif. cont on $[0, b]$

5 If $f_n \rightarrow f$ uniformly on $\xi \in [a, b]$ etc see notes

If $g_n(x) = x^n + 1$ then if $0 \leq x < 1$

$\lim_{n \rightarrow \infty} g_n(x) = \lim_{x \rightarrow \infty} (x^n + 1) = \lim_{n \rightarrow \infty} x^n + 1 = 0 + 1 = 1$

at $\lim_{n \rightarrow \infty} g_n(1) = \lim_{n \rightarrow \infty} (1^n + 1) = 2$

If $f(x)$ is the pointwise limit of g_n

then $f(x)$ is discontinuous (at $x=1$).

Since each $g_n(x)$ is continuous, if the convergence was uniform then the limit fun. would be continuous. But it is not. \therefore the convergence is not unif.

It is unif on $[0, c]$ $\forall 0 < c < 1$.

