UNIVERSITY OF WAIKATO
Department of Mathematics

MATH252A – Elements of Analysis and Algebra

TEST 1

Thursday 29 March 2007 - (55 mins) – Answer ALL questions

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1. (a) Describe $\left\{x \in \mathbb{R} : \dfrac{x+1}{x-1} > 2\right\}$ as an interval.

   (b) If $|x-3| < 1$ show $|x+2| < 6$ $\left[\text{Hint: } |x-3+5|\right]$.

2. Give definitions for each of the following:

   (a) $\lim_{n\to\infty} a_n = L,\ L \in \mathbb{R}$,

   (b) $\lim_{n\to\infty} a_n = \infty$,

   (c) $\sum_{n=1}^{\infty} a_n = s, \quad s \in \mathbb{R}$,

   (d) $S \subset \mathbb{R}$ and $\alpha = \ell ub(S)$.

3. Estimate the value of the following limits and sums as real numbers or $\pm\infty$. There is no need to give proofs.

   (a) $\lim_{n\to\infty} \dfrac{3n^3 + n^2 + 1}{2n^3 + n + 1}$,

   (b) $\lim_{n\to\infty} \dfrac{\sin(n)}{n^2 + 1}$,

   (c) $\sum_{n=1}^{\infty} \left(\dfrac{1}{3^n} + \dfrac{1}{n}\right)$,

   (d) $\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n}$.

4. If $a_n = \dfrac{n-1}{n+1}$ is a sequence, use ε and N_ε to prove that $\lim_{n\to\infty} a_n = 1$ by, given $\varepsilon > 0$, finding an explicit N_ε. Then prove this result by using limit theorems.

5. (a) Use Abel's Test or the Integral Test to show $\sum_{n=1}^{\infty} \dfrac{3}{4n+1}$ diverges.

   (b) Use a comparison and D'Alembert's (Ratio) Test to show $\sum_{n=1}^{\infty} \dfrac{1}{2^n + n}$ converges.

   (c) Use the Alternating Series Test to show $\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{\sqrt{n}}$ converges, and then show it is **not** absolutely convergent.