UNIVERSITY OF WAIKATO
Department of Mathematics

MATH252A – Elements of Analysis and Algebra - 2006

TEST 1

Wednesday 5 April 2006 - (55 mins) – Answer **ALL** questions

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1. Give definitions of the following limits and sums.

(a) $\lim_{n\to\infty} a_n = L$, (b) $\sum_{n=1}^{\infty} b_n = S$,

(c) $\lim_{x\to a} f(x) = L$, (d) $\lim_{x\to a+} f(x) = \infty$.

2. Estimate the value of the following limits and sums as real numbers or $\pm\infty$. There is no need to give proofs.

(a) $\lim_{n\to\infty} \dfrac{n^3+2n+3}{n^2+8n+6}$, (b) $\lim_{n\to\infty} \dfrac{(-1)^n \sin(n)}{n}$,

(c) $\sum_{n=1}^{\infty} \left(\dfrac{1}{2^n}+\dfrac{1}{5^n}\right)$, (d) $\lim_{x\to 2} \dfrac{x^2+6x-3}{x^2+x+4}$,

(e) $\lim_{x\to 1-} \dfrac{x^2-1}{(x-1)^2}$, (f) $\lim_{x\to\infty} \sqrt{x^2+5x+1}-\sqrt{x^2+1}$.

3. If $a_n = \dfrac{7n+2}{n+1}$ is a sequence, use ε and N_ε to prove that $\lim_{n\to\infty} a_n = 7$ by, given $\varepsilon > 0$, finding an explicit N_ε. Then prove this result by using limit theorems.

4. Let $f(x) = 2x + \dfrac{1}{x} + 3$. Given $\varepsilon > 0$, find $\delta_\varepsilon > 0$ so that $|x-1| < \delta_\varepsilon \Rightarrow |f(x) - f(1)| < \varepsilon$. Hence show f is continuous at $x = 1$. Then use $f'(x)$ to show f is continuous on $\mathbb{R}\setminus\{0\}$. Discuss the behaviour of $f(x)$ near $x = 0$ and as $x \to \pm\infty$. Sketch the graph of f.