

The University of Waikato  
Department of Mathematics

Introduction to Real Analysis math252-04A 2004 Assignment 2  
(Corrected)

Due Monday 24th May: Please hand back your completed assignment through the slot for this paper outside the Mathematics Office G3.19.

It should be written up neatly and on no more than four sides of an A4 page or the equivalent.

1. Assuming  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$  use limit theorems to evaluate

$$\lim_{x \rightarrow 0^+} \frac{2x}{\sin x} + \left(\frac{\sin 2x}{x}\right)^2.$$

2. Let  $f, g : (a, b) \rightarrow \mathbb{R}$  be two differentiable functions with  $f'(x) = g'(x)$  for all  $x$  with  $a < x < b$ . Use the Mean Value Theorem to prove there is a constant  $C$  such that  $f(x) = g(x) + c$  for all  $x$  in  $(a, b)$ .

3. Construct a differentiable function  $f$  on  $\mathbb{R}$  such that  $f(x) = 2$  when  $x \leq 0$  and  $f(x) = 0$  when  $x \geq 1$ . (Hint: you will need a cubic polynomial.) Sketch its graph and then apply the Mean Value Theorem to the function on  $[0, 1]$  to find the point at which the slope of the chord equals the slope of the graph of  $f$ .

4. Let a function be defined by

$$f(x) = \frac{x+2}{x+1}$$

Given  $\varepsilon > 0$  find a  $\delta_\varepsilon > 0$  such that

$$|f(x) - 2| < \varepsilon.$$

for all  $x$  with  $0 < x < \delta_\varepsilon$ .

Hence prove that  $f$  is continuous on the right at  $x = 0$ . Sketch the graph of  $f$  and write down all the limiting relationships you can to describe its local and global behaviour.

Kevin Broughan  
13th May 2004