Math 252-03A Intro. to Real Analysis

Assignment 2 Solutions

1
$$\lim_{x\to 1-}\left[\frac{(x^2+1)\cos(\pi x)}{x} + 2\,\frac{x^2-1}{x-1}\right]$$

$$= \lim_{x\to 1-}\frac{(x^2+1)\cos(\pi x)}{x} + 2\lim_{x\to 1-}\frac{(x-1)(x+1)}{x-1} \quad (\text{lt of a sum} = \text{sum of lts})$$

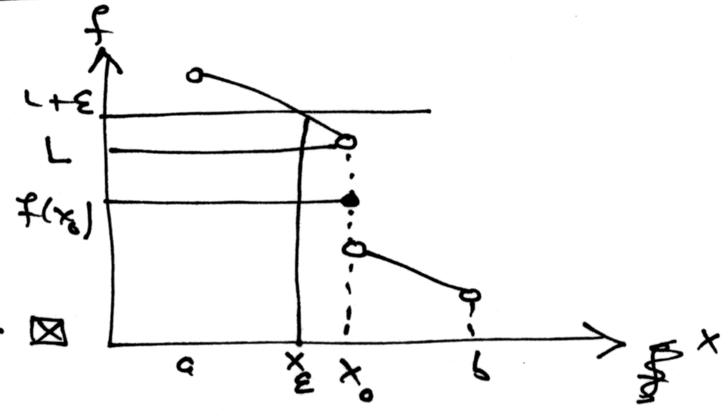
$$= \frac{\lim_{x\to 1-}(x^2+1)\lim_{x\to 1-}\cos(\pi x)}{\lim_{x\to 1-} x} + 2\lim_{x\to 1-}(x+1) \quad (\text{lt of a quot} = \text{quot of lts})$$

$$\left(\frac{(x-1)(x+1)}{x-1} = x+1 \text{ if } x \neq 1\right)$$

$$= \frac{(1^2+1)\cos(\pi)}{1} + 2(1+1) \quad (\text{functions are continuous})$$

$$= 2(-1)+4 = 2 \;//$$

2



Let $L = \text{glb}\{f(x) : x < x_0\}$ $\boxtimes$ and $\varepsilon > 0$ be given. Then for some $x_\varepsilon < x_0$, $\because$ L is the glb,

$$L \leq f(x_\varepsilon) < L + \varepsilon$$

If $x_\varepsilon \leq x < x_0$ $\quad f(x_\varepsilon) \geq f(x) \geq f(x_0)$ since f is decreasing

Hence $\quad L \leq f(x) \leq f(x_\varepsilon) < L + \varepsilon$

Also $x < x_0 \Rightarrow f(x) \geq f(x_0)$ so $f(x_0)$ is a l.b for the set of values $\boxtimes$, so it must be $f(x_0) \leq L$ since L is the glb. ⊛

Therefore $\quad x_0 < x \leq x_\varepsilon \Rightarrow |f(x) - L| = f(x) - L \leq f(x_\varepsilon) - L < \varepsilon$

& so $\lim_{x\to x_0-} f(x) = L \geq f(x_0)$ from ⊛.

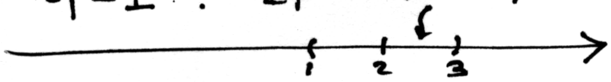
3. $f(x) = \dfrac{2x-1}{x+3}$ given $\varepsilon > 0$

We want $\left| f(x) - \frac{3}{5} \right| < \varepsilon$ this is true

$$\Leftrightarrow \left| \frac{2x-1}{x+3} - \frac{3}{5} \right| < \varepsilon \Leftrightarrow \left| \frac{10x-5-3x-9}{5(x+3)} \right| < \varepsilon$$

$$\Leftrightarrow \frac{|7x-14|}{5|x+3|} < \varepsilon \Leftrightarrow \frac{|x-2|}{|x+3|} < \frac{5\varepsilon}{7}$$

let $\delta_1 = 1$. If $|x-2| < 1 = \delta_1$ then $-1 < x-2 < 1$



so $1 < x < 3$

$\Rightarrow 4 < x+3 < 6$

$\therefore |x+3| \geq 4$

If we have $\dfrac{|x-2|}{4} < \dfrac{5\varepsilon}{7}$ then $\dfrac{|x-2|}{|x+3|} \leq \dfrac{|x-2|}{4} < \dfrac{5\varepsilon}{7}$

so let $\delta_\varepsilon = \min\left\{1, \dfrac{20\varepsilon}{7}\right\}$.

This shows $\lim_{x \to 2} f(x) = \frac{3}{5}$ But $f(2) = \dfrac{2.2-1}{2+3} = \dfrac{3}{5}$

Hence $\lim_{x\to 2} f(x) = f(2) \Rightarrow f$ is continuous at $x = 2$.

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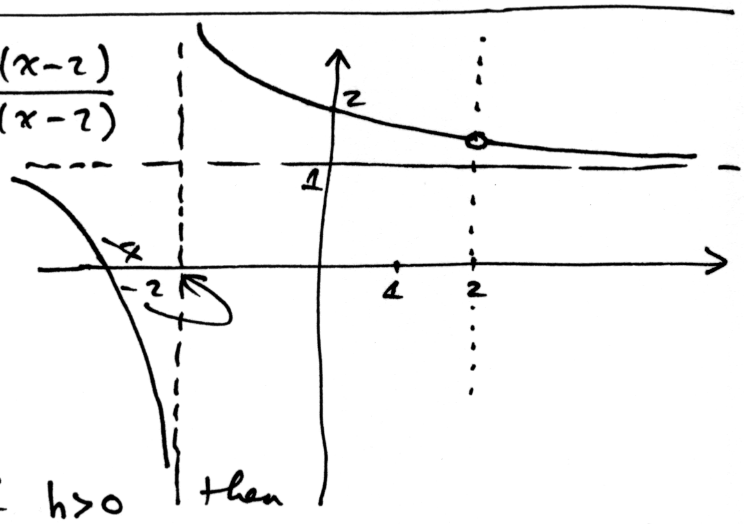
4. $f(x) = \dfrac{x^2+2x-8}{x^2-4} = \dfrac{(x+4)(x-2)}{(x+2)(x-2)}$



for $x \neq 2$ $f(x) = \dfrac{x+4}{x+2} \Rightarrow f(0) = 2$.

and $f(x) = 0 \Rightarrow x+4 = 0 \Rightarrow x = -4$

Plotting the graph, the value @ $x = 2$ must be missing.

At $x = -2$, $f(x)$ is undefined. If $h > 0$ then

$$f(-2+h) = \frac{-2+h+4}{-2+h+2} = \frac{h+2}{h} = 1 + \frac{2}{h} \text{ so } \lim_{x\to -2+} f(x) = \lim_{h\to 0+} f(x)$$

and $\displaystyle\lim_{x\to -2-} f(x) = \lim_{h\to 0+} \frac{-2-h+4}{-2-h+2} = \lim_{h\to 0+} \frac{2-h}{-h}$ $\displaystyle= \lim_{h\to 0+}\left(1+\frac{2}{h}\right) = +\infty$.

$x = -2-h$ $\displaystyle= \lim_{h\to 0+}\left(1 - \frac{2}{h}\right) = -\infty$.

when x is large $\displaystyle\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{1+4/x}{1+2/x} = \frac{1+0}{1+0} = 1$. //