The University of Waikato
Department of Mathematics

Introduction to Real Analysis math252-03A 2003 Assignment 2

Due Thursday 15th May: Please hand back your completed assignment through the slot for this paper outside the Mathematics Office G3.19.

It should be written up neatly and on no more than four sides of an A4 page or the equivalent.

1. Use limit theorems to evaluate

$$\lim_{x\to 1-}\left[\frac{(x^2+1)\cos\pi x}{x}+2\frac{x^2-1}{x-1}\right].$$

2. Let $f:(a,b)\to R$ be a monotonely decreasing function. Prove that if x_o is any point with $a<x_o<b$ that

$$\lim_{x\to x_o-} f(x)\geq f(x_o).$$

3. Let a function be defined by

$$f(x)=\frac{2x-1}{x+3}$$

Given $\varepsilon>0$ find an $\delta_\varepsilon>0$ such that

$$\left|f(x)-\frac{3}{5}\right|<\varepsilon.$$

for all x with $0<|x-2|<\delta_\varepsilon$.
Hence prove that f is continuous at $x=2$.

4. Explore the limiting behaviour of the function defined below first informally after sketching its graph and /or using a calculator and then using limit theorems and finally through proving at least two of the limits using $\varepsilon/\delta_\varepsilon$. The limits to develop are $x\to 0+, x\to 2, x\to -2+, x\to\infty$.

$$f(x)=\frac{x^2+2x-8}{x^2-4}$$

Kevin Broughan
8th May 2003