The University of Waikato
Department of Mathematics

Introduction to Real Analysis math252-04A 2004 Assignment 1

Due Thursday 25th March: Please hand back your completed assignment through the slot for this paper outside the Mathematics Office G3.19.

It should be written up neatly and on no more than four sides of an A4 page or the equivalent.

1. Let a, b be real numbers. Prove that $||a| - |b|| \leq |a - b|$.

2. Use limit theorems to prove that

$$\lim_{n\to\infty}(\frac{n+1}{2n})(1-\frac{1}{n})=\frac{1}{2}.$$

3. Use the sandwich theorem to prove that

$$\lim_{n\to\infty}(\frac{(-1)^n}{n^2+\sin n})=0.$$

4. Let a sequence (a_n) be defined by

$$a_n = \frac{4n+1}{n+6}$$

Given $\varepsilon > 0$ find an $N_\varepsilon \in N$ such that

$$|a_n - 4| < \varepsilon$$

for all n with $N_\varepsilon \leq n$.
Hence prove that $a_n \to 4$.

5. Explore, using a calculator, some values of the sequence (a_n) where

$$a_n = \sqrt{n^2+2n+3} - \sqrt{n^2+n+1}$$

Make an estimate for whether you believe the limit exists and what its value might be. Then try to prove your estimate is correct by transforming the expression for the sequence and manipulating the transformed expression until limit theorems can be applied. Kevin Broughan
18th March 2004