The University of Waikato
Department of Mathematics

Introduction to Real Analysis math252-03A 2003 Assignment 1

Due Thursday 27th March: Please hand back your completed assignment through the slot for this paper outside the Mathematics Office G3.19.

It should be written up neatly and on no more than four sides of an A4 page or the equivalent.

1. Let $S = \{2 - \frac{1}{n^2} : n = 1, 2, 3, \dots\}$**. Prove that**

$$2 = \text{lub S}, \quad 1 = \text{glb S}.$$

2. Use limit theorems to prove that

$$\lim_{n\to\infty} \frac{2n^2 - n + 1}{n^2 + 6} + \frac{4n + 1}{n} = 6.$$

3. Use the sandwich theorem to prove that

$$\lim_{n\to\infty}\left(\frac{\sin(n) + n^2}{n^2}\right) = 1.$$

4. Let a sequence (a_n) **be defined by**

$$a_n = \frac{2n + 1}{n - 1}$$

Given $\varepsilon > 0$ **find an** $N_\varepsilon \in N$ **such that**

$$|a_n - 2| < \varepsilon$$

for all n **with** $N_\varepsilon \leq n$.
Hence prove that $a_n \to 2$.

5. If $0 < c < 1$ **and** $a_n = c^n$ **for** $n = 1, 2, \dots$**, prove that** $a_n \to 0$**. Hint: write** $a_{n+1} = c \cdot a_n$ **and use a monotone convergence theorem.**

Kevin Broughan
20 March 2003