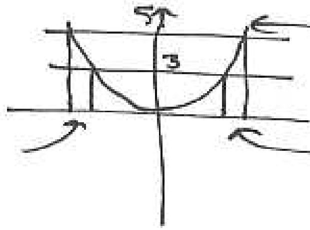
1) $|x^2-4|<1 \iff -1<x^2-4<1 \iff 3<x^2<5$ (*)

$x>0 \iff \sqrt{3}<x<\sqrt{5}$ or

$x<0 \quad \sqrt{3}<-x<\sqrt{5} \iff -\sqrt{5}<x<-\sqrt{3}$

so $\{x : |x^2-4|<1\} = (-\sqrt{5},-\sqrt{3}) \cup (\sqrt{3},\sqrt{5})$

OR by (*)



2) $$\lim_{n\to\infty}\left(\frac{n+1}{3n+2}\right)\left(6+\frac{1}{n^2}\right) = \lim_{n\to\infty}\frac{n+1}{3n+2}\cdot\lim_{n\to\infty}\left(6+\frac{1}{n^2}\right) \quad (a)$$

$$= \lim_{n\to\infty}\frac{1+\frac{1}{n}}{3+\frac{2}{n}}\cdot\left(\lim_{n\to\infty}6+\left(\lim_{n\to\infty}\frac{1}{n}\right)^2\right) \quad (b)$$

$$= \frac{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)}{\lim_{n\to\infty}\left(3+\frac{2}{n}\right)}\left(6+0^2\right) \quad (c) \quad = \left(\frac{1+0}{3+0}\right)(6+0) = \frac{6}{3} = 2 \;//$$

(a) lt. of a prod. is the prod. of the limits

(b) ÷ numerator and denominator by n, lt. of a sum is the sum of the lts.

(c) lt. of a quotient is the quotient of the lts.

3) $\forall n: \; -1 \le \sin(n) \le 1 \implies -1 \le (-1)^n\sin(n) \le 1$ also

Hence $\dfrac{-1}{n^2} \le \dfrac{(-1)^n\sin(n)}{n^2} \le \dfrac{1}{n^2}$ since $n^2>0$

$\implies \quad 2-\dfrac{1}{n^2} \le \dfrac{(-1)^n\sin(n)}{n^2}+2 \le 2+\dfrac{1}{n^2} \quad \forall n\in\mathbb{N}$. But $2-\frac{1}{n^2} \to 0 \leftarrow 2+\frac{1}{n^2}$

$\implies$, by the Sandwich Thm, $\dfrac{(-1)^n\sin(n)}{n^2}+2 \to 2$ also.

4) Given $\varepsilon > 0$. Want $|a_n - \frac{1}{2}| < \varepsilon$. This is true

$$\Leftrightarrow \left|\frac{n+1}{2n+1} - \frac{1}{2}\right| < \varepsilon$$

$$\Leftrightarrow \left|\frac{2n+2-2n-1}{2(2n+1)}\right| < \varepsilon$$

$$\Leftrightarrow \frac{1}{2(2n+1)} < \varepsilon$$

$$\Leftrightarrow 2(2n+1) > \frac{1}{\varepsilon}$$

$$\Leftrightarrow 2n+1 > \frac{1}{2\varepsilon} \Leftrightarrow n > \frac{1}{4\varepsilon} - \frac{1}{2}$$

so let N_ε be any element of $\mathbb{N}$ with $N_\varepsilon > \frac{1}{4\varepsilon} - \frac{1}{2}$. So if $n \geq N_\varepsilon$ we have $n \geq N_\varepsilon > \frac{1}{4\varepsilon} - \frac{1}{2} \Rightarrow n > \frac{1}{4\varepsilon} - \frac{1}{2}$ which is equivalent to $|a_n - \frac{1}{2}| < \varepsilon$. Hence $a_n \to 1/2$ //

) $a_n = \dfrac{2^n - 1}{2^n} = \dfrac{2^n}{2^n} - \dfrac{1}{2^n} = 1 - \dfrac{1}{2^n}$.

$$a_{n+1} > a_n \Leftrightarrow 1 - \frac{1}{2^{n+1}} > 1 - \frac{1}{2^n}$$

$$\Leftrightarrow \frac{1}{2^n} > \frac{1}{2^{n+1}} = \frac{1}{2^n} \cdot \frac{1}{2}$$

$$\Leftrightarrow 1 > \frac{1}{2}$$ which is true. Hence (a_n) is increasing.

lso (a_n) is bounded above by 1.

$$a_n = 1 - \frac{1}{2^n} < 1 \Leftrightarrow -\frac{1}{2^n} < 0 \Leftrightarrow 0 < \frac{1}{2^n}$$ which is true.

Hence $\lim_{n\to\infty} a_n \ \exists$.

To calculate the limit:
$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(1 - \frac{1}{2^n}\right) = 1 - \lim_{n\to\infty}\left(\frac{1}{2}\right)^n = 1 - 0 = 1$$

nce from the v. useful limits $c^n \to 0$ when $|c| < 1$ //