**The University of Waikato**
**Department of Mathematics**

**Elements of Analysis and Algebra math252-07A 2007 Assignment 1**

**Due Thursday 15th March: Please hand back your completed assignment through the slot for this paper outside the Mathematics Office G3.19.**

**It should be written up neatly and on no more than four sides of an A4 page or the equivalent.**

**1. Express the set $\{x : |x^2-4| < 1\}$ as the union of two open intervals OR (harder) prove that $\sqrt{3}$ is not a rational number and generalize to $\sqrt{n}$ for $n \in \mathbb{N}$ being as large a set as possible.**

**2. Use limit theorems to prove that**

$$\lim_{n\to\infty} (\frac{n+1}{3n+2})(6+\frac{1}{n^2})=2.$$

**3. Use the sandwich theorem to prove that**

$$\lim_{n\to\infty}(\frac{(-1)^n \sin n}{n^2}+2) = 2.$$

**4. Let a sequence $(a_n)$ be defined by**

$$a_n = \frac{n+1}{2n+1}$$

**Given $\varepsilon > 0$ find an $N_\varepsilon \in \mathbb{N}$ such that**

$$\left|a_n - \frac{1}{2}\right| < \varepsilon$$

**for all $n$ with $N_\varepsilon \leq n$.**
**Hence prove that $a_n \to \frac{1}{2}$.**

**5. Let the sequence $(a_n)$ be defined by $a_n = (2^n-1)/2^n$. Prove that the sequence is increasing and bounded above. Deduce that the limit exists. Find the limit using the limit theorem.**

**Kevin Broughan 8th March 2007**