The University of Waikato
Department of Mathematics

Introduction to Real Analysis math252-06A 2006 Assignment 1

Due Thursday 16th March: Please hand back your completed assignment through the slot for this paper outside the Mathematics Office G3.19. (Neatly and on no more than four sides of an A4 page).

1. Find all the real numbers x such that $\frac{1}{1-x} > \frac{1}{2}$.
2. Let $\epsilon > 0$. Show that $|x| \leq \epsilon$ implies $-\epsilon \leq x \leq \epsilon$.

3. Use the limit theorem for sequences to prove that

$$\lim_{n\to\infty}(\frac{n^2+1}{n^2+2})(2-\frac{1}{n})=2,$$

stating at each step which part of the theorem you are using.

4. Use the sandwich theorem to prove that

$$\lim_{n\to\infty}(\frac{n\sin(n)}{n^2+1})=0.$$

5. Let a sequence (a_n) be defined by

$$a_n = \frac{6n-2}{2n+1}$$

Given $\varepsilon > 0$ find an $N_\varepsilon \in N$ such that

$$|a_n - 3| < \varepsilon$$

for all n with $N_\varepsilon \leq n$.
Hence prove that $a_n \to 3$.

6. Let $S = \{\frac{n}{n+1} | n \in \mathbf{N}\}$. Prove that $\mathrm{glb}S = 1/2$ and the $\mathrm{lub}S = 1$.
Kevin Broughan

13th March 2006