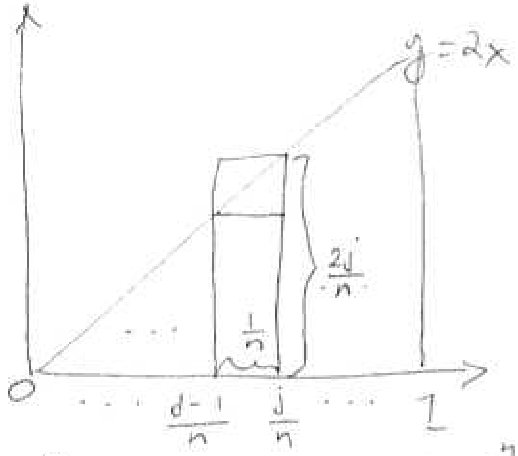
math252-09 B Assign 4

(1) $T = (2,3)$ then $\sup(-T) = \sup((-3,-2)) = -2$
$\inf(T) = \inf((2,3)) = 2 \Rightarrow$

$\Rightarrow \quad -2 = \sup(-T) = -\inf(T)$ //

(2)



$$L(f,P_n) = \sum_{j=1}^{n} \frac{2(j-1)}{n}\cdot\frac{1}{n} = \frac{2}{n^2}\sum_{j=1}^{n} j - \sum_{j=1}^{n}\frac{2}{n^2} = \frac{2}{n^2}\frac{n}{2}(n+1) - \frac{2}{n^2}\cdot n$$

$$= \frac{n+1}{n} - \frac{2}{n} = 1 + \frac{1}{n} - \frac{2}{n} = 1 - \frac{1}{n}$$

and $$U(f,P_n) = \sum_{j=1}^{n} \frac{2j}{n}\cdot\frac{1}{n} = \frac{2}{n^2}\sum_{j=1}^{n} j = \frac{2}{n^2}\frac{n}{2}(n+1) = \frac{n+1}{n} = 1 + \frac{1}{n}$$

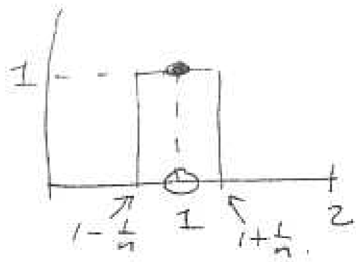
$$U(f,P_n) - L(f,P_n) = \left(1+\frac{1}{n}\right) - \left(1 - \frac{1}{n}\right) = \frac{2}{n} = \varepsilon_n \to 0 \quad \therefore f \text{ is Ri.In.}$$

Hence

$$L(f,P_n) \le \int_0^1 f \le U(f,P_n) \Rightarrow 1 - \frac{1}{n} \le \int_0^1 f \le 1 + \frac{1}{n}. \text{ Let } n\to\infty$$

$$\Rightarrow 1 + 0 \le \int_0^1 f \le 1 + 0 \Rightarrow \int_0^1 f = 1$$ //

(3)



$P_n = (0, 1-\frac{1}{n}, 1+\frac{1}{n}, 2)$

$$L(f,P_n) = 0\cdot\Delta x_1 + 0\cdot\Delta x_2 + 0\cdot\Delta x_3 = 0$$

$$U(f,P_n) = 0\cdot\Delta x_1 + 1\left(1+\frac{1}{n} - \left(1 - \frac{1}{n}\right)\right) + 0\cdot\Delta x_3 = \frac{2}{n}$$

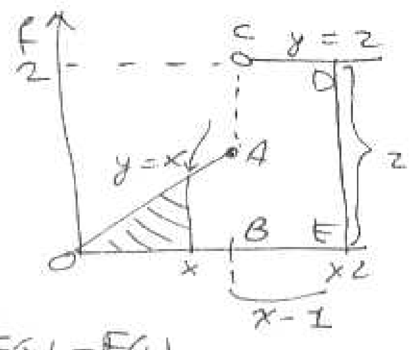
Hence $U(f,P_n) - L(f,P_n) = \frac{2}{n} \to 0 \Rightarrow$

f is Ri Int

$0 \le \int_0^2 f \le \frac{2}{n} \Rightarrow$, let $n \to \infty$ so $\int_0^2 f = 0$. //

(4) If $0 \le x \le 1$, $F(x) = \int_0^x f = \frac{1}{2} x \cdot x = \frac{x^2}{2}$

If $1 < x$, $F(x) = \text{area } \Delta OAB + \text{rect } BCDE$
$= \frac{1}{2} + 2(x-1)$



$$\Rightarrow F(x) = \begin{cases} \frac{x^2}{2}, & x \le 1 \\ 2x - \frac{3}{2}, & x > 1 \end{cases} \Rightarrow F'(x) = \begin{cases} x & x < 1 \\ 2 & x > 1 \end{cases}$$

$$\lim_{x\to1-} F(x) = \lim_{x\to1-} \frac{x^2}{2} = \frac{1}{2} = \lim_{x\to1+}\left(2x - \frac{3}{2}\right) = \lim_{x\to1+} F(x) = F(1).$$ //