) $f(x) = x(x-1)(x-2) = x(x^2-3x+2) = x^3-3x^2+2x$

$f(x) = x^3-3x^2+2x$

$f'(x) = 3x^2-6x+2$

$f''(x) = 6x-6$

$f(0+h) = f(0) + f'(0)h + \frac{1}{2!}f''(0+\theta h)h^2$

$h^3-3h^2+2h = 0 + 2h + \frac{1}{2!}(6\theta h-6)h^2$

$\Rightarrow \quad h^3-3h^2 = \frac{6}{2}(\theta h-1)h^2$

$h \neq 0 \quad h-3 = 3(\theta h-1) \quad \Rightarrow \quad \theta = \frac{1}{3}$ ie. $|\theta| = \frac{1}{3} < 1$. $\forall h \neq 0$

$\therefore$ fr $h = 0.1$.

---

5) $f(a+h) = f(a) + f'(a)h + \frac{f''(a)h^2}{2!} + \frac{f'''(a)h^3}{3!} + \frac{f^{(iv)}(a+\theta h)h^4}{4!}$

since

$f'''(x) = 6$ & $f''''(x) = f^{(iv)}(x) = 0 \quad \Rightarrow \quad f^{(iv)}(a+\theta h) = 0$

$\Rightarrow \quad f(a+h) = f(a) + f'(a)h + \frac{f''(a)h^2}{2!} + \frac{f'''(a)h^3}{3!}$

Explicitly $\quad LHS = (a+h)^3 - 3(a+h)^2 + 2(a+h)$

$= a^3+3a^2h+3ah^2+h^3 - 3(a^2+2ha+h^2) - 2(a+h)$

$= a^3+3a^2h+3ah^2+h^3 - 3a^2 - 6ah - 3h^2 - 2a - 2h$

$= a^3+3a^2h+3ah^2+h^3$

RHS $= f(a) \quad = a^3-3a^2+2a \quad = a^3-3a^2+2a$

$+ f'(a)h \quad + h(3a^2-6a+2) \quad + 3a^2h - 6ah + 2h$

$+ \frac{f''(a)h^2}{2!} \quad + \frac{6h^2}{2}(a-1) \quad + 3ah^2 - 3h^2$

$+ \frac{f'''(a)h^3}{3!} \quad + \frac{6h^3}{6}(1) \quad + h^3$

and $\quad LHS = RHS$.

(6)

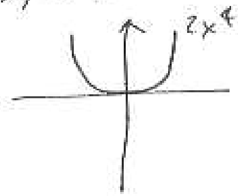
A point of inflection is where the tangent to a curve crosses the curve i.e. on one side it is below and on the other above the curve.

Here $f(x) = x^3 - 3x^2 + 2x \implies f''(x) = 6x - 6, \; f'''(x) = 6$

Then $f''(1) = 0$ and $f'''(1) = 6 > 0$ so the first non vanishing derivative, other than the first, is of odd order $\implies x = 1, y = f(1)$ is a point of inflection. Since $f^{(iv)}(x) = 0 \; \forall x \in \mathbb{R}$ there are no higher order pts of inflection. Note that $f'(1) = 3 \cdot 1^2 - 6.1 + 2 \neq 0$, so the pt. of inflection is not horizontal.

(7) Let $g(x) = x^4(x-1)(x-2)$

Near $x = 0$ $\quad g(x) \approx x^4(0-1)(0-2) = 2x^4$ so $g(x)$ is like



$\therefore \; x = 0$ give a local minimum.

OR

$$g(x) = x^4(x^2 - 3x + 2) = x^6 - 3x^5 + 2x^4 \implies g(0) = 0$$
$$g'(x) = 6x^5 - 3x^4 + 8x^3 \implies g'(0) = 0$$
$$g''(x) = 30x^4 - 12x^3 + 24x^2 \implies g''(0) = 0$$
$$g'''(x) = 4.30x^3 - 3.12x^2 + 24.2x \implies g'''(0) = 0$$
$$g^{(iv)}(x) = 3.4.30x^2 - 2.3.12x + 2.24 \implies g^{(iv)}(0) = 48 > 0$$

since the first non vanishing derivative is positive $\implies$ local min.

Taylor Series

$$g(x) = x^6 - 3x^5 + 2x^4$$
$$= 0 + 0.x + 0.x^2 + 0.x^3 + 2x^4 + (-3)x^5 + x^6 + 0 + 0$$

is the Taylor series for $g(x)$.