252-09B Assign 1 Solutions

(1.) If $x-1 \geq 0$ then $x+1 < |x-1|$
$\Leftrightarrow x+1 < x-1$
$\Leftrightarrow 1 < -1 \Leftrightarrow 2 < 0$ which is false

$\therefore$ there are no solutions ~~in~~ with $x-1 \geq 0 \Leftrightarrow x \geq 1 \Leftrightarrow x \in [1,\infty)$

If $x-1<0$ then $x+1 < |x-1| \Leftrightarrow x+1 < -(x-1)$
$\Leftrightarrow x+1 < -x+1$
$\Leftrightarrow 2x < 0$
$\Leftrightarrow x < 0 \Leftrightarrow x \in (-\infty,0)$

and $x-1<0 \Leftrightarrow x<1 \Leftrightarrow x \in (-\infty,1)$

$\therefore$ the solution set is $(-\infty,0)\cap(-\infty,1) = (-\infty,0)$.

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(2) If $a<b<0$ then $a<0$ and $b<0 \Rightarrow ab>0 \Rightarrow \frac{1}{ab}>0$

Hence $\frac{1}{ab}a < \frac{1}{ab}b \Rightarrow \frac{1}{b} < \frac{1}{a}$.

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(3) (a) Clearly 2 is a l.b. for S. Given $\varepsilon < 2$ there is an element of $x \in S$ (say $x = 2+\frac{\varepsilon}{2}$) with $2 \leq x < \varepsilon$. Hence $2 = \text{glb}(S)$.

(b) Clearly 5 is an u.b. for S. If there was a smaller upper bound $b<5$ then, since $5 \leq b$ cannot be true ($5 \leq b < 5 \Rightarrow 5<5$), such a b cannot exist. Therefore 5 is the least upper bound.

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(4) Let $A \subset B$ and $\text{lub}(B) = \beta$. Then $\forall x \in A$, $x \in B \Rightarrow x \leq \beta$, since the lub is an u.b. Hence β is an u.b for A. But then it must sat $\text{lub}(A) \leq \beta = \text{lub}(B)$, since any particular u.b is $\geq$ the lub. //
