MATHEMATICS WITH CALCULUS - USEFUL FORMULAE AND TABLES

ALGEBRA

uadratics

If $ax^2+bx+c=0$ then $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

omplex Numbers

$z=x+iy$
$=r(\cos\theta+i\sin\theta)$
$=r\,\mathrm{cis}\,\theta$

$|z|=\sqrt{(x^2+y^2)}=r$

$\arg z=\theta$ where $\cos\theta=\dfrac{x}{r}$ and $\sin\theta=\dfrac{y}{r}$

De Moivre's Theorem: If $n\in I$ then $(r\ \mathrm{cis}\ \theta)^n=r^n\ \mathrm{cis}(n\,\theta)$

garithms

If $y=b^x$ then $\log_b y=x$

If $y=e^x$ then $x=\ln y=\log_e y$

$\log_b x+\log_b y=\log_b xy$

$\log_b x-\log_b y=\log_b \dfrac{x}{y}$

$\log_b x^n=n\log_b x$

$\log_b x=\dfrac{\log_a x}{\log_a b}$

SERIES

ithmetic

$=a+(n-1)d$. $S_n=\dfrac{n}{2}[2a+(n-1)d]$

Geometric

$t_n=ar^{n-1}$. $S_n=\dfrac{a(1-r^n)}{1-r}$, $r\neq 1$. $S_\infty=\dfrac{a}{1-r}$ for $|r|<1$

ponential

$=1+x+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\cdots$ (for all x)

Logarithmic

$\ln(1+x)=x-\dfrac{x^2}{2}+\dfrac{x^3}{3}-\dfrac{x^4}{4}+\cdots$ (for $-1<x\leq 1$)

nomial Theorem

$+b)^n=\binom{n}{0}a^n+\binom{n}{1}a^{n-1}b^1+\binom{n}{2}a^{n-2}b^2+\cdots+\binom{n}{r}a^{n-r}b^r+\cdots+\binom{n}{n}b^n$

$\binom{n}{r}={}^nC_r=\dfrac{n!}{(n-r)!\,r!}$

ome values of $\binom{n}{r}$ are given in the table below.

inomial Coefficients

n \ r	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1287	1716	1716	1287	715	286
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003

COORDINATE GEOMETRY

ircle

$(x-a)^2+(y-b)^2=r^2$.
has centre (a,b) and radius r

Ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ or $(a\cos\theta,\ b\sin\theta)$

arabola $y^2=4ax$ or $(at^2,\ 2at)$

Hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ or $(a\sec\theta,\ b\tan\theta)$

Identities

$\cos^2\theta + \sin^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \mathrm{cosec}^2\theta$

Compound Angles

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

Double Angles

$\sin 2A = 2\sin A\cos A$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

Products

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Sums

$\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

$\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\cos D - \cos C = 2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

General Solutions

If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n\alpha$

If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$

If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$

where $n \in \{\text{integers}\}$

CALCULUS

Differentiation

$y = f(x)$	$\dfrac{dy}{dx} = f'(x)$
c	0
x^n	nx^{n-1}
$\ln x$	$\dfrac{1}{x}$
e^{ax}	ae^{ax}
a^x	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x\tan x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x\cot x$
$\cot x$	$-\mathrm{cosec}^2 x$

Product Rule

$(f.g)' = f'.g + f.g'$

or if $y = uv$ then $\dfrac{dy}{dx} = \dfrac{du}{dx}v + u\dfrac{dv}{dx}$

Quotient Rule

$\left(\dfrac{f}{g}\right)' = \dfrac{g.f' - f.g'}{g^2}$

or if $y = \dfrac{u}{v}$ then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Composite Function Rule

$f(g(x))' = f'(g(x)).g'(x)$

or if $y = f(u)$ and $u = g(x)$ then $\dfrac{dy}{dx} = \dfrac{dy}{du}.\dfrac{du}{dx}$

NUMERICAL METHODS

Trapezium Rule

$\int_a^b f(x)\,dx \approx \frac{1}{2}h[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})]$ where $h = \dfrac{b-a}{n}$ and $y_r = f(x_r)$.

Simpson's Rule

$\int_a^b f(x)\,dx \approx \frac{1}{3}h[y_0 + y_n + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2})]$ where $h = \dfrac{b-a}{n}$, $y_r = f(x_r)$ and n is even

Euler's Method

$y_{n+1} = y_n + h.y'_n$ $\qquad x_{n+1} = x_n + h$