Part B

1 (a) Let $u = x^2+1 \Rightarrow du = 2x\,dx \Rightarrow$

$$\int 2x(x^2+1)^4 dx = \int (x^2+1)^4 2x\,dx$$
$$= \int u^4 du = \frac{u^5}{5} + C$$
$$= \frac{(x^2+1)^5}{5} + C$$

(b) $$\int \frac{x}{\sqrt{1-x^2}} dx = \int \frac{du}{\sqrt{u}}$$ $\Leftarrow$ $u = 1-x^2$, $-\frac{1}{2} du = x\,dx$

$$= -\frac{1}{2}\int u^{-\frac{1}{2}} du$$
$$= -\frac{1}{2} 2u^{\frac{1}{2}} + C$$
$$= -u^{1/2} + C = -\sqrt{1-x^2} + C$$

(c) Let $u = y+1 \Rightarrow du = dy$ & $y = u-1$

$$\int \frac{y}{\sqrt{y+1}} dy = \int \frac{u-1}{\sqrt{u}} du = \int \left(\frac{u}{u^{1/2}} - \frac{1}{u^{1/2}}\right) du$$
$$= \int (u^{1/2} - u^{-1/2}) du = \frac{2u^{3/2}}{3} - 2u^{1/2} + C$$
$$= \frac{2}{3}(y+1)^{3/2} - 2(y+1)^{1/2} + C$$

(d) $$\int \sin^3(x) dx = \int \sin^2(x) \sin(x)\, dx$$ $\begin{cases} u = \cos x \\ -du = \sin(x) dx \end{cases}$ $\Leftarrow$

$$= -\int (1-u^2) du$$
$$= -\left(u - \frac{u^3}{3}\right) + C$$
$$= \frac{\cos^3 x}{3} - \cos(x) + C .$$

2 (a) $$f(x) = \frac{x(x^2+1)^2}{x+2} \Rightarrow \ln|f(x)| = \ln\left(\frac{|x||x^2+1|^2}{|x+2|}\right)$$
$$= \ln|x| + 2\ln|x^2+1| - \ln|x+2|$$

$$\Rightarrow \frac{f'(x)}{f(x)} = (\ln|x|)' + 2(\ln|x^2+1|)' - (\ln|x+2|)'$$
$$= \frac{1}{x} + \frac{2.2x}{x^2+1} - \frac{1}{x+2} \Rightarrow$$ (next page)

$$f'(x) = f(x)\left[\frac{1}{x} + \frac{4x}{x^2+1} - \frac{1}{x+2}\right]$$

$$= \frac{x(x^2+1)^2}{x+2}\left[\frac{1}{x} + \frac{4x}{x^2+1} - \frac{1}{x+2}\right]$$

This is not defined for $x = 0$ or -2, but multiplying out the bracket we get

$$f'(x) = \frac{(x^2+1)^2}{x+2} + \frac{4x^2(x^2+1)^1}{x+2} - \frac{x(x^2+1)^2}{(x+2)^2}$$

& this is not defined only when $x = -2$.

Finally $$f'(1) = \frac{1(2^2)}{3}\left[\frac{1}{1} + \frac{4}{2} - \frac{1}{3}\right]$$

$$= \frac{32}{9}$$

b) $$\int_1^2 x^2 \ln(x)\,dx = \int_1^2 \underbrace{(\ln x)}_{u}\,\underbrace{x^2\,dx}_{dv}$$

Let:
$$\frac{dv}{dx} = x^2 \Rightarrow v = \frac{x^3}{3}$$
$$u = \ln x \Rightarrow$$
$$\frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{dx}{x}$$

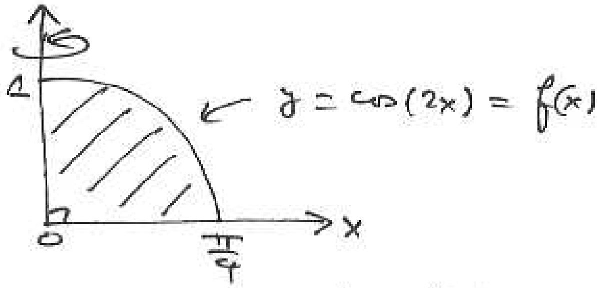
$$= uv\Big|_1^2 - \int_1^2 v\,du$$

$$= \frac{x^3}{3}\ln x\Big|_1^2 - \int_1^2 \frac{x^3}{3}\frac{1}{x}\,dx$$

$$= \frac{2^3}{3}\ln 2 - 0 - \frac{1}{3}\int_1^2 x^2\,dx = \frac{2^3}{3}\ln 2 - \frac{1}{9}x^3\Big|_1^2$$

$$= \frac{8}{3}\ln 2 - \frac{7}{9}$$

c) $$\text{Vol} = 2\pi\int_0^{\pi/4} x\,f(x)\,dx.$$



$$= 2\pi\int_0^{\pi/4} \underbrace{x}_{u}\,\underbrace{\cos(2x)\,dx}_{dv}$$

Let: $u = x \Rightarrow du = dx$

$\frac{dv}{dx} = \cos(2x) \Rightarrow v = \frac{1}{2}\sin(2x)$

$$= 2\pi\left(uv\Big|_0^{\pi/4} - \int_0^{\pi/4} v\,du\right)$$

$$= 2\pi\left(\frac{x}{2}\sin(2x)\Big|_0^{\pi/4} - \int_0^{\pi/4}\frac{1}{2}\sin 2x\,dx\right)$$

$$= 2\pi\left(\frac{\pi}{8}\cdot 1 - 0 - \left(-\frac{1}{4}\cos(2x)\right)\Big|_0^{\pi/4}\right.$$

$$= \frac{\pi^2}{4} + \frac{2\pi}{4}\left(\cos(\pi/2) - \cos(0)\right) = \frac{\pi^2}{4} - \frac{\pi}{2}.$$