UNIVERSITY OF WAIKATO
Department of Mathematics

○ = Correct
◌ = close to correct

MATH101B - Introduction to Calculus

TEST 2

Tuesday 2 October 2007 - 7.05 pm **Time Allowed: 1 hour**

Part A - Answer questions on the *ANSWER SHEET* provided.
This is worth **48%** of the total marks and you should not spend more than about half the time on it.

Part B - Answer the **2 questions** in order. This is worth **52%** of the total marks.

No one is to leave the lecture room **during the last 15 minutes** of the test period.

Calculators (**NOT** programmable) may be used. Two pages of formulas are available.

PART A

MULTI-CHOICE (each question is worth 4%)

1. $\displaystyle\int_0^1 6(x+5)^4 dx =$

(a) 5^5 (b) $\dfrac{6^6}{5}$ (c) 1

(d) $6^6 - 5^5$ (e) None of these.

2. Which of the following is an antiderivative for $3e^{-3x} - \sin 2x$?

(a) $\cos(2x) + e^{-3x}$ (b) $\cos(2x) - 9e^{-3x}$ (c) $e^{-3x} - \dfrac{\cos(2x)}{2}$

(d) $\cos^2(x) - e^{-3x}$ (e) $\dfrac{\cos(2x)}{2} - \dfrac{e^{-3x}}{3}$.

3. $\displaystyle\int_0^1 (x^2 + 2x)/\sqrt{x}\, dx =$

(a) 26/15 (b) 9/7 (c) 13/15

(d) 26/5 (e) None of these.

4. The expression $\sum_{j=1}^{n} 3\overline{x}_j(1-\overline{x}_j)\Delta x_j$ represents a Riemann sum for the integral of a function $f(x)$.

The function is

(a) $3x - x^3$ (b) $3x(1-x)\,dx$ (c) $\int_0^x f(t)\,dt$

(d) $3x - 3x^2$ (e) None of these.

5. If $F(x) - \sin(x) = \int_0^x (1+t)^2\,dt$ then $F'(0) =$

(a) -1 (b) 0 (c) 1

(d) 2 (e) None of these.

6. The length of arc between $x = a$ and $x = b$ of the curve $y = f(x)$, where f and f' are continuous on $[a, b]$, is

(a) $\pi \int_a^b (f(x))^2\,dx$ (b) $2\pi \int_a^b x f(x)\,dx$ (c) $2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$

(d) $\int_a^b \sqrt{1+(f'(x))^2}\,dx$ (e) None of these.

7. Which of the following identities is **NOT** correct?

(a) $\cos(2x) = 2\cos^2(x) + 1$ (b) $\cos(2x) = 1 - 2\sin^2 x$ (c) $1 + \tan^2(x) = \sec^2(x)$

(d) $\sin^2(x) + \cos^2(x) = 1$ (e) $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$.

8. Which of the following statements is **NOT** correct?

(a) $y = 2\cos(x)$ satisfies the equation $y'' + y = 0$ (b) $y = 2\sin(x)$ satisfies the equation $y'' + y = 0$

(c) $y = 2\tan(x)$ satisfies the equation $y' = 2(1+y^2)$ (d) $y = 2\ln(x)$ satisfies the equation $y' = 2/x$

(e) $y = 2e^x$ satisfies the equation $y' - y = 0$.

9. If $f:[2,\infty]\to\mathbb{R}$ is defined by $f(x)=4-4x+x^2$ then $f^{-1}(x)$ is

(a) undefined (b) $+\sqrt{x}+2$ (c) $-\sqrt{x}+2$

(d) $1/(4-4x+x^2)$ (e) None of these.

10. If $\int e^{-x}\cos(x)dx = Ae^{-x}\cos x + Be^{-x}\sin x + C$ then

(a) $A=-1/2$ and $B=1/2$ (b) $A=-1/2$ and $B=-1/2$ (c) $A=1/4$ and $B=-1/4$

(d) $A=-1$ and $B=0$ (e) None of these.

11. The volume of the solid generated when the region under the curve $f(x)=8x(1-x)$ between $x=0$ and $x=1$ is revolved about the line $y=1$ is given by

(a) $\int_0^1 \pi f(x)^2 dx$ (b) $\int_0^1 \pi f(x-1)^2 dx$ (c) $\int_0^1 \pi(f(x)-1)^2 dx$

(d) $\int_0^1 \pi(f(x)^2-1^2)dx$ (e) None of these.

12. Find the value of the integral $\int_0^{\pi/2} \frac{\cos(x)}{1+\sin^2(x)}dx$

(a) 0 (b) $\pi/4$ (c) $\pi/2$

(d) $3\pi/4$ (e) None of these.

PART B

(Each question is worth 26%)

1. Find expressions for the following indefinite integrals

(a) $\int 2x(x^2+1)^4\,dx$

(b) $\int \frac{x}{\sqrt{1-x^2}}\,dx$

(c) $\int \frac{y}{\sqrt{y+1}}\,dy$

(d) $\int \sin^3(x)dx.$

2. (a) Use logarithmic differentiation to find an expression for the derivative of

$$f(x)=\frac{x(x^2+1)^2}{(x+2)}$$

Simplify your expression and note that values of x for which it is well defined. Compute $f'(1)$.

(b) Evaluate $\int_1^2 x^2\ln(x)dx.$

(c) The area under the graph of the function $f(x)=\cos(2x)$ from $x=0$ to $x=\pi/4$ is revolved about the $y-$axis. Use the method of shells to calculate the volume of the solid of revolution.