UNIVERSITY OF WAIKATO
**Department of Mathematics**

# *MATH101-08A - Introduction to Calculus*

## TEST 1

**Wednesday 2 April 2008** **Time Allowed: 1 hour**

**Part A** - Answer questions on the *ANSWER SHEET* provided.
This is worth **50%** of the total marks and you should not spend more than about half the time on it.

**Part B** - Answer the **2** questions in any order. This is worth **50%** of the total marks.

**No one is to leave** the lecture room **during the last 10 minutes** of the test period.

Calculators (**NOT programmable**) may be used.

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## PART A
## *MULTI-CHOICE*

◯ = correct or best answer
◌ = near answer

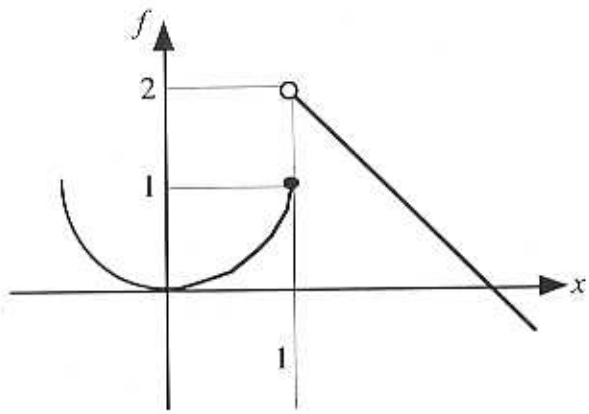
*(Each question is worth 5%. Answer on the sheet provided.)*

1. The domain of the function $f(x)=\dfrac{\sqrt{x}}{x-2}$ is

(A) $\mathbb{R}$ (B) $[0,\infty)$ (C) $\mathbb{R}\setminus\{2\}$

(D) $[0,\infty)\setminus\{2\}$ (E) $(-\infty,0)\cup\{2\}$.

2. For the function $f(x)$ with the given graph, which of the following statements is **NOT** correct?




(A) $\lim_{x\to 1-} f(x)=1$ (B) $\lim_{x\to 1-} f(x)=f(1)$

(C) $\lim_{x\to 1+} f(x)=2$ (D) $f$ is differentiable at $x=1$

(E) $f$ is continuous and differentiable at every point except $x=1$.

3. Let

$$f(x)=\begin{cases}2 & , \quad x<1\\ 1 & , \quad x=1\\ x^2+1, & x>1.\end{cases}$$

Which of the following statements is **NOT** correct?

(A) $\lim_{x\to 1+} f(x)=2$ (B) $\lim_{x\to 1} f(x)$ does exist (C) $\lim_{x\to 1} f(x)=f(1)$

(D) $f$ is **not** continuous on $\mathbb{R}$ (E) $f'_+(1)=0$.

4. $\lim_{\theta\to 0}\dfrac{\tan(2\theta)}{\theta}$

(A) 2 (B) $\dfrac{1}{2}$ (C) 1

(D) 0 (E) None of these.

5. The slope of the graph of $y=x^2+3x+7$ at $(-1, 5)$ is

(A) 1 (B) $2x+3$ (C) 5

(D) $\dfrac{dy}{dx}$ (E) None of these.

6. The tangent to the curve $y=\dfrac{x+1}{x-1}$ at (2, 3) has equation

(A) $y=\dfrac{x+1}{x-1}$ (B) $-2$ (C) $y=7-2x$

(D) $y=2x+7$ (E) None of these.

7. If $f(x) = x^{3/4} - \frac{1}{2}x^{-2}$ then for $x > 0$, $f'(x) =$

(A) $\frac{3}{4}x^{7/4} + x^{-1}$ (B) $\frac{3}{4}x^{-1/4} - x^{-3}$ (C) $\frac{3}{4}\frac{1}{x^{1/4}} + \frac{1}{x^3}$

(D) $\frac{3}{4x^{1/4}} - \frac{1}{x^3}$ (E) None of these.

8. The slope of the tangent to the curve $4x - xy + y^3 = 7$ at the point $(2, 1)$ is

(A) $-3$ (B) $3$ (C) $\frac{y-4}{3y^2 - x}$

(D) Does not exist (E) None of these.

9. If $f(x) = (2x+3)\sin(x^2 + 3x + 1)$ then $f'(x) =$

(A) $-\cos(x^2 + 3x + 1)$ (B) $\sin(x^2 + 3x + 1)(4x^2 + 12x + 11)$

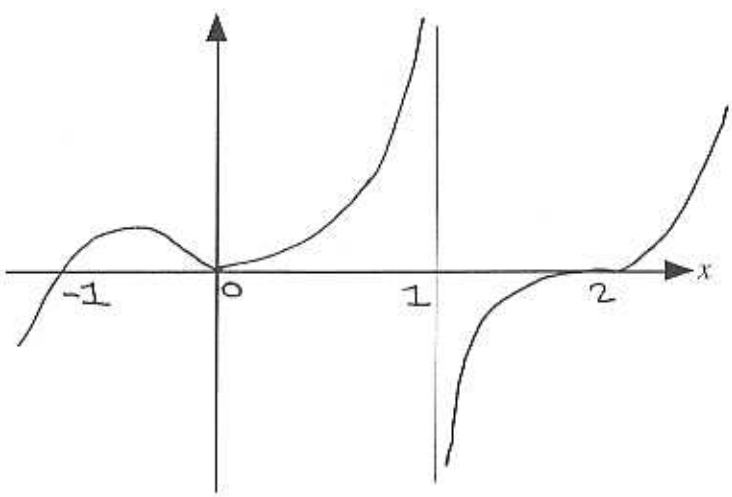
(C) $2\cos(x^2 + 3x + 1) + (2x+3)^2 \sin(x^2 + 3x + 1)$

(D) $-2\cos(x^2 + 3x + 1)$ (E) None of these.

10. The sketched graph is most like the graph of the rational function

(A) $\frac{(x+1)^2 x(x-2)^7}{x-1}$ (B) $\frac{(x+1)(x^2)(x-2)^2}{(x-1)^2}$ (C) $\frac{(x+1)x^2(x-2)^3}{(x-1)^2}$

(D) $\frac{(x+1)(x^2)(x-2)^3}{(x-1)}$ (E) None of these.



# PART B

*(Each question is worth 25%)*

1. (a) Explain briefly what the statements "$f$ is continuous at $x=a$" and "$f$ is differentiable at $x=a$" mean. Give an example of a function which is continuous but not differentiable at $x=1$.

(b) Let $f(x)=\begin{cases} x^2 & f \quad x \le 1 \\ ax+b & f \quad x>1. \end{cases}$

Find values for the constants $a$ and $b$ so that $f(x)$ is continuous and differentiable at $x=1$.

2. (a) Use the limit definition of the derivative $f'(a)$ to find the derivative of $f(x)=x^3+2x^2+1$ at $x=a$.

Hint: Use $(a+h)^3=a^3+3a^2h+3ah^2+h^3$.

(b) A girl flies a kite at a height of 300 ft. The wind carries the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?