

Part A

PART A

Questions	1	2	3	4	5	6	7	8	9	10	11	12	13
Answers	(d)	(c)	(d)	(e)	(b)	(d)	(b)	(e)	(c)	(b)	(b)	(e)	(d)

Part B

1 (a) By $\lim_{x \rightarrow a^+} f(x) = L$ we mean we can make $f(x)$

"arbitrarily close to L provided we choose $x > a$ and sufficiently close to a ."

By " f is continuous at $x = a$ " we mean $\lim_{x \rightarrow a} f(x) = f(a)$.

(b) $\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} (2) = 2$

$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} (ax+b) = a(-1)+b = -a+b$

$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (ax+b) = a(3)+b = 3a+b$

$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (-2) = -2$

Here we need $2 = -a+b$ ① and.

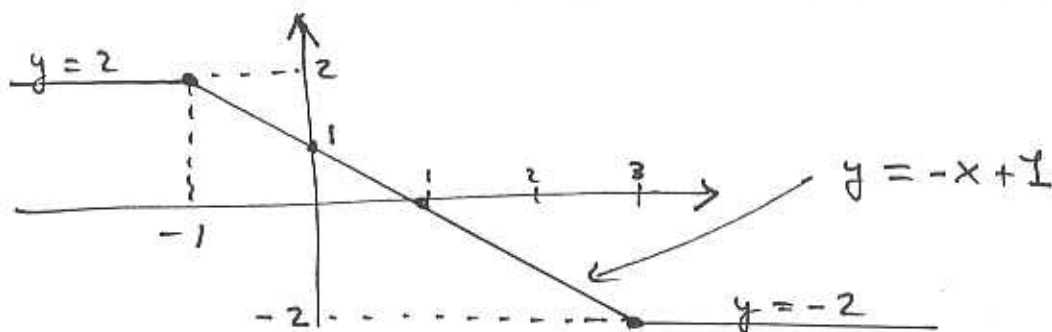
$-2 = 3a+b$ ②

$\Leftrightarrow 2 = -3a-b$ ③

① + ③ $\Rightarrow 4 = -4a \Rightarrow a = -1$, then ① \Rightarrow

$b = 2+a = 1$

so $\boxed{a = -1, b = 1}$



(2) (a) f'(a) = lim_{h to 0} (f(a+h) - f(a)) / h

f(x) = (2+x)/x => f(1) = 3

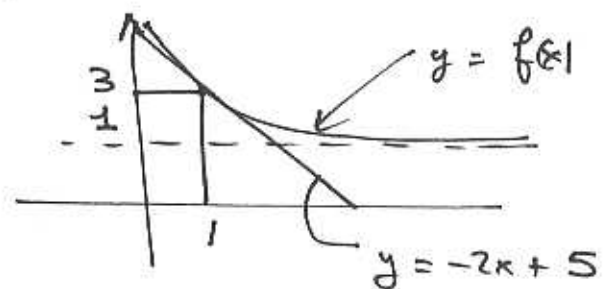
f'(1) = lim_{h to 0} (f(1+h) - f(1)) / h = lim_{h to 0} ((2+1+h)/(1+h) - (2+1)/1) / h = lim_{h to 0} (3+h - 3(1+h)) / (h(1+h)) = lim_{h to 0} (-2h) / (h(1+h)) = -2

(b) Eqn of the tangent is -2 = (y-3)/(x-1)

=> y - 3 = -2(x-1) or y = -2x + 2 + 3 = -2x + 5

On (0, infinity), x > 0 so f(x) > 0. f(x) = 2/x + x/x = 1 + 2/x

as x -> 0+ f(x) -> 1 + infinity = infinity
as x -> infinity f(x) -> 1 + 0 = 1 Hence



Note that f'(x) = -2/x^2 < 0 for all x > 0, so there are no critical points.

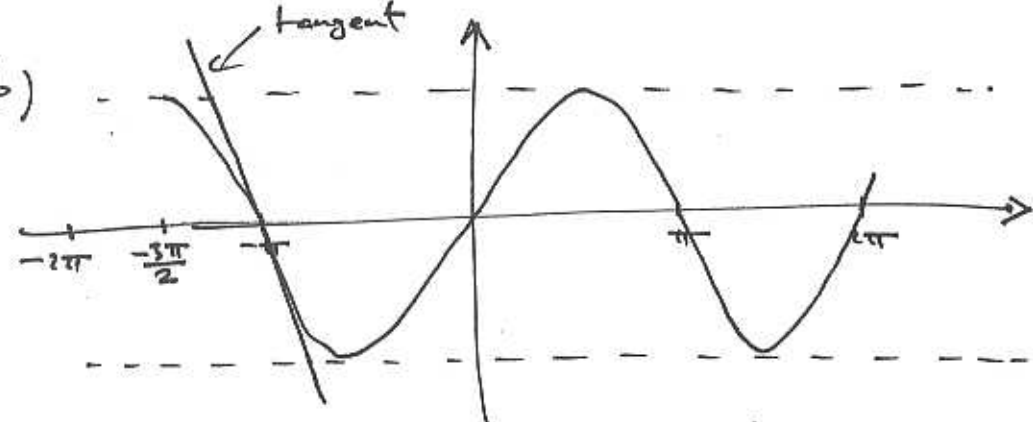
(3) (a) y = 4(cos(x))^-1 + (tan(x))^-1 =>

dy/dx = -4(cos(x))^-2 * (-sin(x)) - (tan(x))^-2 * sec^2(x) = 4 * (sin(x)/cos^3(x)) - sec^2(x)/tan^2(x)

But tan(pi/4) = 1 and sec(pi/4) = 1/cos(pi/4) = sqrt(2) =>

at x = pi/4: dy/dx = 4 * (1/2) - 2/1 = -2 + 4*sqrt(2) //

3 (b)



$$f(x) = \sin(x) \Rightarrow f'(x) = \cos(x) \quad f'(-\pi) = \cos(-\pi) = \cos(2\pi - \pi) = \cos(\pi) = -1$$

So eqn is $-1 = \frac{y - 0}{x - (-\pi)}$ since $\sin(-\pi) = \sin(\pi) = 0$

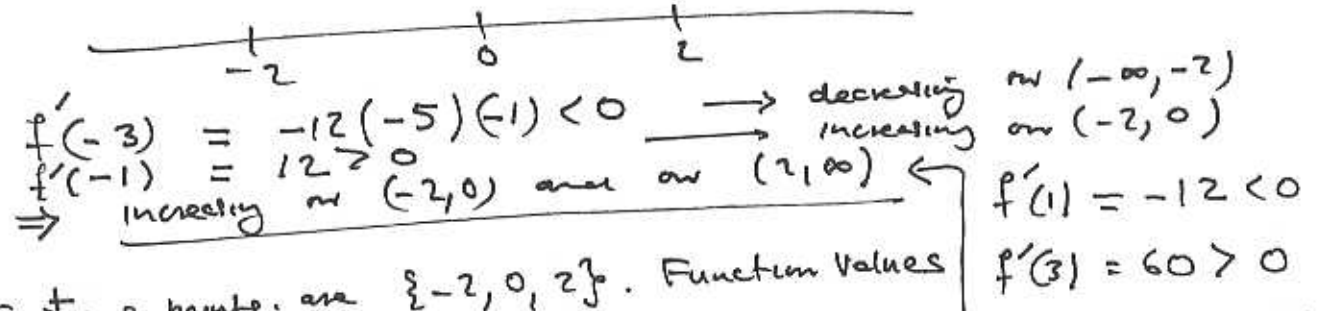
$$\Rightarrow y = -(x + \pi) = -x - \pi //$$

4 (a) $f(x) = x^4 - 8x^2 + 16$ on $[-3, 2]$

$$f'(x) = 4x^3 - 16x = 4x(x^2 - 4) = 4x(x-2)(x+2)$$

$$f''(x) = 12x^2 - 16$$

$f'(x) > 0 \Leftrightarrow f(x)$ is increasing. $f(x) = 0 \Leftrightarrow x \in \{-2, 0, 2\}$



(b) Critical points: are $\{-2, 0, 2\}$. Function values
 $f(-2) = 0, f(0) = 16, f(2) = 0$
Singular points: there are none.
End points $f(-3) = 25, f(2) = 0$

(c) at $x = -3, f'(x) < 0 \Rightarrow$ local maxima,
 at $x = -2, f''(x) = 32 > 0 \Rightarrow$ local minima,
 at $x = 0, f''(x) = -16 < 0 \Rightarrow$ local maxima,
 at $x = 2, f''(x) = 32 > 0 \Rightarrow$ local minima

(d) The values of the function are $\{0, 16, 0, 25, 0\}$
 Hence the global minimum is 0 and global maximum 25.
 at $(2 \text{ and } -2)$ (at -3)